MAIN ASSOCIATES

31 August, 1999

Materials & Test Section
Texas Department of Transportation
125 East 11th St.
Austin, TX 78701-2483

Attention: Wei Wu

Subject: The Relationship Between Stiffness & Modulus

Enclosure: Poulos, H.G., and Davis, E.H., *Elastic Solutions For Soil & Rock Mechanics*, 1974, pages 167-168.

Dear Wei,

Equation 7.16 on page 168 of the enclosure documents the subject relationship that Humboldt uses with its stiffness gauge. Be careful, as the nomenclature is confusing in the text. The terms in the equation are as follows.

P = force
E = Young's modulus
R_2 = Outside radius of ring (2.25" for the stiffness gauge)
υ = Poisson's ratio
ρ_z = displacement

This can be confirmed by checking the units of the equation.

The value of $\omega(n)$ used for the stiffness gauge is determined by plotting the values in Table 7.1 and interpolating (see fig, 1).

The form of the equation that Humboldt uses follows directly from equation 7.16.

$$K = \frac{P}{\rho_z} \sim \frac{1.77 R_2 E}{(1-\upsilon^2)}$$

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I also have some questions regarding the work that the stiffness gauges is associated with.

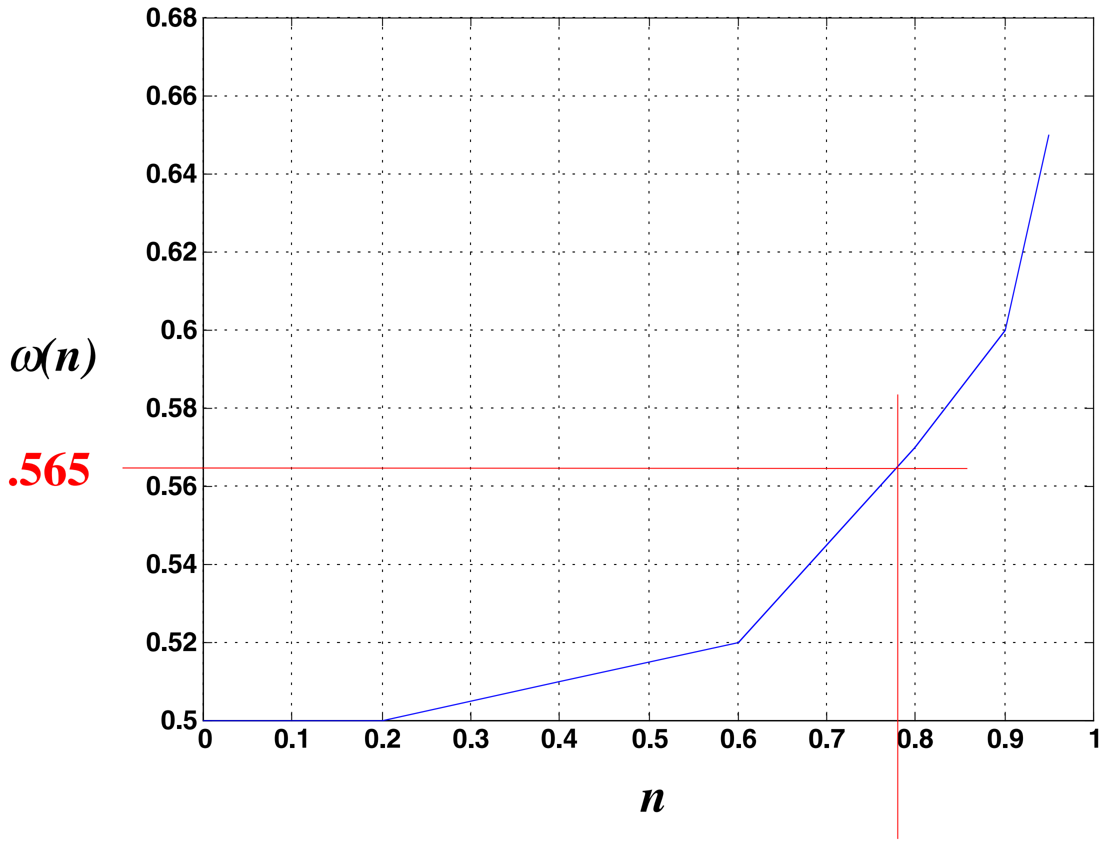
- If I remember correctly, your ultimate goal is to move to modulus for evaluating and controlling soil compaction. Were are you in this transition? What would it take for TXDOT to adopt modulus in place of density?
- Does TXDOT plan to implement some type of process control for soil compaction?
- If the stiffness gauge is to be adopted by TXDOT for the field measurement of modulus or for use in controlling the compaction process, in your opinion, what criteria must it meet?
- In your recent report, you assigned values of modulus to levels of base quality. Can you give me some insight into how those assignments were made?
- In you work with the stiffness gauge, were any laboratory measurements of resilient modulus made, per AASHTO T-292, that might be compared to the modulus derived from stiffness gauge measurements? Does TXDOT place any importance on laboratory measurements of resilient modulus?

Sincerely,

Melvin Main

Cc:
Scott Fiedler

Figure 1



Stiffness Gauge: R_1/R_2 = 1.75"/2.25" = .777

ELASTIC SOLUTIONS FOR SOIL AND ROCK MECHANICS

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Borowicka (1943) obtained the following solution for the rotation ϕ:

$$\phi = \frac{3M(1-\nu^2)}{4Ea^3} \quad \ldots (7.10)$$

Sneddon (1946) obtained the following solution for vertical stress σ_z in the mass:

$$\sigma_z = \frac{3M}{4a^3\pi}\left[R^{-\frac{1}{2}}\sin\tfrac{1}{2}\psi + \frac{z}{a}R^{-3/2}\left\{\frac{z}{a}\sin(\tfrac{3}{2}\psi) - \cos(\tfrac{3}{2}\psi)\right\}\right] \quad \ldots (7.11)$$

where $R^2 = \left\{(\frac{r}{a})^2+(\frac{z}{a})^2-1\right\}^2+4(\frac{z}{a})^2$

$$\psi = \tan^{-1}\left\{\frac{2\frac{z}{a}}{(\frac{r}{a})^2+(\frac{z}{a})^2-1}\right\}$$

For the particular case of the contact pressure beneath the base,

$$\sigma_z = \frac{3M}{4a^3\pi}\frac{1}{\sqrt{1-(\frac{r}{a})^2}} \qquad (0 \le \frac{r}{a} \le 1) \quad \ldots (7.12)$$

On the axis at depth z,

$$\sigma_z = \frac{3M}{4a^3\pi}\left[\frac{1+3(\frac{z}{a})^2}{\{1+(\frac{z}{a})^2\}^2}\right] \quad \ldots (7.13)$$

The complete distribution of stress, strain and displacement may be obtained from the results in Appendix B.

7.2.4 TORSION LOADING (Fig.7.5)

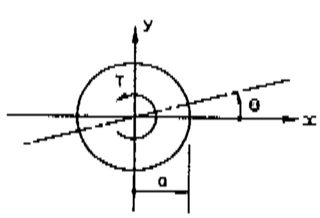

FIG.7.5

Reissner and Sagoci (1944) give the following solution for angular rotation θ:

$$\theta = \frac{3T(1+\nu)}{8Ea^3} \quad \ldots (7.14)$$

The complete distribution of stress, strain and displacement may be obtained from the solutions given in Appendix B.

7.3 Circular Ring on Semi-Infinite Mass

This problem has been considered by Egorov (1965). (1965).

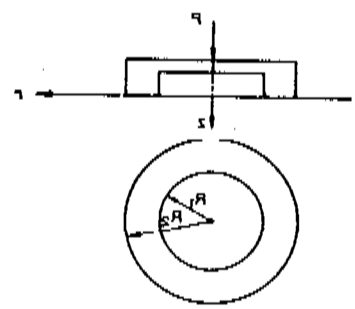

FIG.7.6

The contact pressure distribution is expressed as follows:

$$p = \frac{p_{av}(1-n^2)\sqrt{(\frac{r}{R_2})^2-m^2}}{2\sqrt{1-m^2}\,E_o(k)\sqrt{\{(\frac{r}{R_2})^2-n^2\}\{1-(\frac{r}{R_2})^2\}}} \quad \ldots (7.15)$$

where $n = \frac{R_1}{R_2}$

$$p_{av} = \frac{P}{\pi R_2^2(1-n^2)}$$

$$E_o(k) = \int_0^{\pi/2}\sqrt{(1-k^2\sin^2\theta)}\,d\theta$$

= complete elliptic integral of second kind

$$k = \left(\frac{1-n^2}{1-m^2}\right)^{\frac{1}{2}}$$

$m = 0.8n$ for $0 \le n \le 0.9$, increasing to 1 for $n=1$ (note that $n=0$ is the case of a circle).

Contact pressure distributions for various values of n are shown in Fig.7.7.

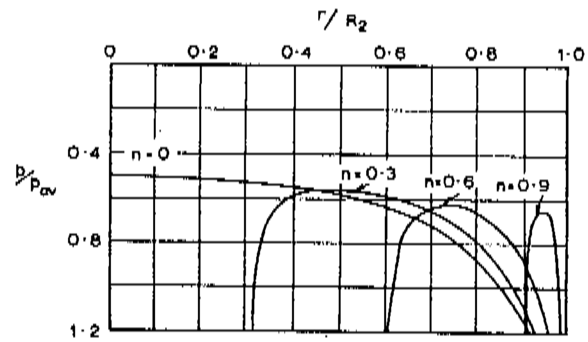

FIG.7.7 Contact pressure beneath rigid circular ring (Egorov, 1965).

It has been found that for $n \geq 0.9$, a rigid ring behaves essentially as a rigid strip foundation (see Section 7.1).

The vertical displacement of a rigid ring may be expressed as

$$\rho_z = \frac{P(1-\nu^2)}{ER_2}\,\omega(n) \qquad \ldots (7.16)$$

Values of $\omega(n)$ are tabulated in Table 7.1.

TABLE 7.1
VALUES OF $\omega(n)$ FOR RIGID RING (Egorov, 1965)

n	0	0.2	0.4	0.6	0.8	0.9	0.95
$\omega(n)$	0.50	0.50	0.51	0.52	0.57	0.60	0.65

7.4 Rectangle on Semi-Infinite Mass

In all cases below, the rectangle is smooth.

7.4.1 SYMMETRICAL VERTICAL LOADING.

The following approximate solution for the vertical displacement ρ_z is quoted by Whitman and Richart (1967):

$$\rho_z = \frac{P(1-\nu^2)}{\beta_z\sqrt{BL}\,E} \qquad \ldots (7.17)$$

where P = total vertical load
B, L = rectangle dimensions
β_z = factor dependent on L/B and plotted in Fig.7.8.

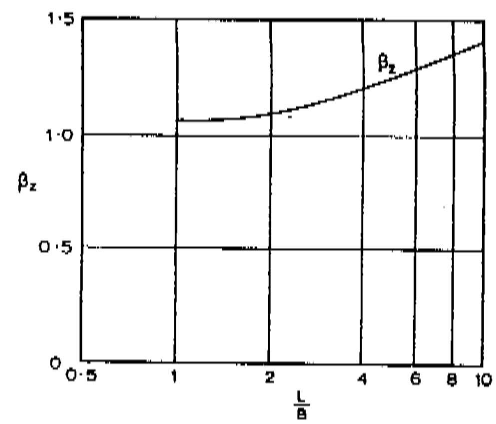

FIG.7.8 Coefficient β_z for rigid rectangle (Whitman and Richart, 1967).

7.4.2 HORIZONTAL LOADING

Barkan (1962) gives the following approximate solution for horizontal displacement ρ_h due to a load Q in the direction of B:

$$\rho_h = \frac{Q(1-\nu^2)}{\beta_x\sqrt{BL}\,E} \qquad \ldots (7.18)$$

β_x is a factor dependent on L/B and ν and is tabulated in Table 7.2.

TABLE 7.2
VALUES OF β_x (Barkan, 1962)

ν	L/B						
	0.5	1	1.5	2	3	5	10
0.1	1.040	1.000	1.010	1.020	1.050	1.150	1.250
0.2	0.990	0.938	0.942	0.945	0.975	1.050	1.160
0.3	0.926	0.868	0.864	0.870	0.906	0.950	1.040
0.4	0.844	0.792	0.770	0.784	0.806	0.850	0.940
0.5	0.770	0.704	0.692	0.686	0.700	0.732	0.840

7.4.3 MOMENT LOADING

Lee (1962) gives the following approximate solution for rotation ϕ of the base of the rectangle due to moment M applied in the direction of B:

$$\phi = \frac{M(1-\nu^2)}{B^2LE}\,I_\theta \qquad \ldots (7.19)$$

I_θ is a function of L/B and is given in Table 7.3.

Solutions quoted by Whitman and Richart (1967) for $L/B<1$ are also given.

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