INTRODUCTION TO SOIL MODULI

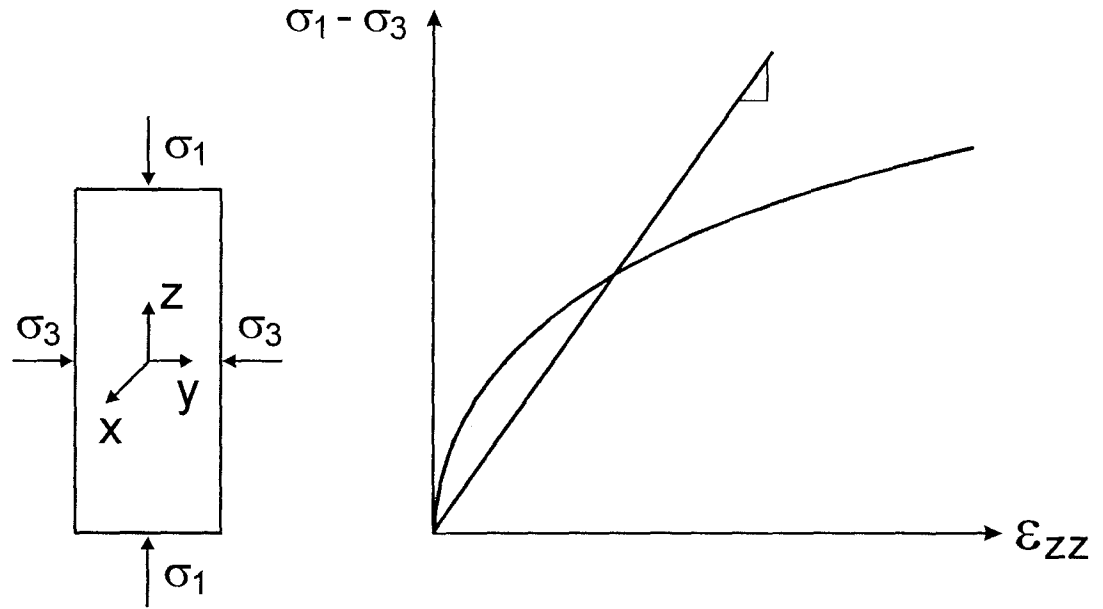by
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INTRODUCTION

The modulus of a soil is one of the most difficult soil parameters to estimate because it depends on so many factors. Therefore when one says for example: "The modulus of this soil is 10,000 kPa", one should immediately ask: "What are the conditions associated with this number?". The following is an introduction to some of the important influencing factors for soil moduli. It is not meant to be a thorough academic discourse but rather a first step in understanding the complex world of soil moduli. In a first part, the modulus is defined. In a second part, the factors influencing the modulus and related to the state of the soil are described. In a third part, the factors related to the loading process are discussed. Fourth, some applications of soil moduli are presented. In a fifth and sixth part, the soil modulus is compared to the soil stiffness and to the soil coefficient of subgrade reaction respectively.

DEFINITION

How does one obtain a modulus from a stress strain curve? In order to answer this question, the example of the stress strain curve obtained in a triaxial test is used. The sample is a cylinder, it is wrapped in an impermeable membrane and confined by an all around (hydrostatic) pressure. Then the vertical stress is increased gradually and the non linear stress strain curve shown on Figure 1 is obtained. Elasticity assumes that the strains experienced by the soil are linearly related to the stresses applied. In reality this is not true for soils and there lies the complexity. The equations of elasticity for this axi-symmetric loading relate the stresses and the strains in the three directions as shown in Figure 1. Because of the axi-symmetry, equations 1 and 2 are identical. In equations 1 and 3 on Figure 1 there are two unknowns: the soil modulus E and the Poisson's ratio v. In the triaxial test, it is necessary to measure the stresses applied in both directions as well as the strains induced in both directions in order to calculate the modulus of the soil. Indeed one needs two simultaneous equation to solve for E and v. Note that **the modulus is not the slope of the stress strain curve**. An exception to this statement is the case where the confining stress is zero as it is for a typical concrete cylinder test or an unconfined compression test on a clay. In order to calculate the Poisson's ratio, it is also necessary to measure the stresses applied in both directions as well as the strains induced in both directions. Note also that **the**

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right) = \frac{1}{E}\left(\sigma_3 - \nu(\sigma_1 + \sigma_3)\right) \qquad (1)$$

$$\varepsilon_{yy} = \frac{1}{E}\left(\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right) = \frac{1}{E}\left(\sigma_3 - \nu(\sigma_1 + \sigma_3)\right) \qquad (2)$$

$$\varepsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right) = \frac{1}{E}\left(\sigma_1 - \nu(\sigma_3 + \sigma_3)\right) \qquad (3)$$

$$E = \frac{\sigma_1 - 2\nu\,\sigma_3}{\varepsilon_{zz}} \qquad (4)$$

$$\frac{\varepsilon_{xx}}{\varepsilon_{zz}} = \frac{\sigma_3 - \nu(\sigma_1 + \sigma_3)}{\sigma_1 - 2\nu\,\sigma_3} \qquad (5)$$

Figure 1. Calculating a Modulus

Poisson's ratio is not the ratio of the strains in both directions (equation 5 on Figure 1). An exception to this statement is again the case where the confining stress is zero.

Which modulus? Secant, tangent, unload, reload, or cyclic modulus? Because soils do not exhibit a linear stress strain curve, many moduli can be defined from the triaxial test results for example. In the previous paragraph, it was pointed out that the slope of the stress strain curve is not the modulus of the soil. However the slope of the curve is related to the modulus and it is convenient to associate the slope of the stress strain curve to a modulus. Indeed this gives a simple image tied to the modulus value; note however that in the figures the slope is never labeled as modulus E but rather as slope S. Referring to Figure 2, if the slope is drawn from the origin to a point on the curve (O to A on Figure 2), the secant slope S_s is obtained and the secant modulus E_s is calculated from it. One would use such a modulus for predicting the movement due to the first application of a load as in the case of a spread footing. If the slope is drawn as the tangent to the point considered on the stress strain curve then the tangent slope S_t is obtained and the tangent modulus E_t is calculated from it. One would use such a modulus to calculate the incremental movement due to an incremental load as in the case of the movement due to one more story in a high-rise building. If the slope is drawn as the line which joins points A and B on Figure 2, then the unloading slope S_u is obtained and the unloading modulus E_u is calculated from it. One would use such a modulus when calculating the heave at the bottom of an excavation or the rebound of a pavement after the loading by a truck tire (resilient modulus). If the slope is drawn from point B to point D on Figure 2, then the reloading slope S_r is obtained and the reload modulus E_r is calculated from it. One would use this modulus to calculate the movement at the bottom of an excavation if the excavated soil or a building of equal weight was placed back in the excavation or to calculate the movement of the pavement under reloading by the same truck tire. If the slope is drawn from point B to point C on Figure 2, then the cyclic slope S_c is obtained and the cyclic modulus E_c is calculated from it. One would use such a modulus and its evolution as a function of the number of cycles for the movement of a pile foundation subjected to repeated wave loading.

Which ever one of these moduli is defined and considered, the state in which the soil is at a given time will affect that modulus. The next section describes some of the main state parameters influencing soil moduli.

STATE FACTORS

The state factors include the following.

How closely packed are the particles? If they are closely packed, the modulus tends to be high. This is measured by the dry density (ratio of the weight of solids over the total volume of the wet sample) of the soil for example; it can also be measured by the porosity (ratio of the volume of voids over the total volume of the wet sample).

How are the particles organized? This refers to the structure of the soil. For example a coarse grain soil can have a loose or dense structure and a fine grain soil can have a dispersed or flocculated structure. Note that two soil samples can have the same dry density yet different structures and therefore different soil moduli. This is why taking a disturbed sample of a coarse grain soil in the field and reconstituting it to the same dry density and water content in the laboratory can lead to laboratory and field moduli which are different.
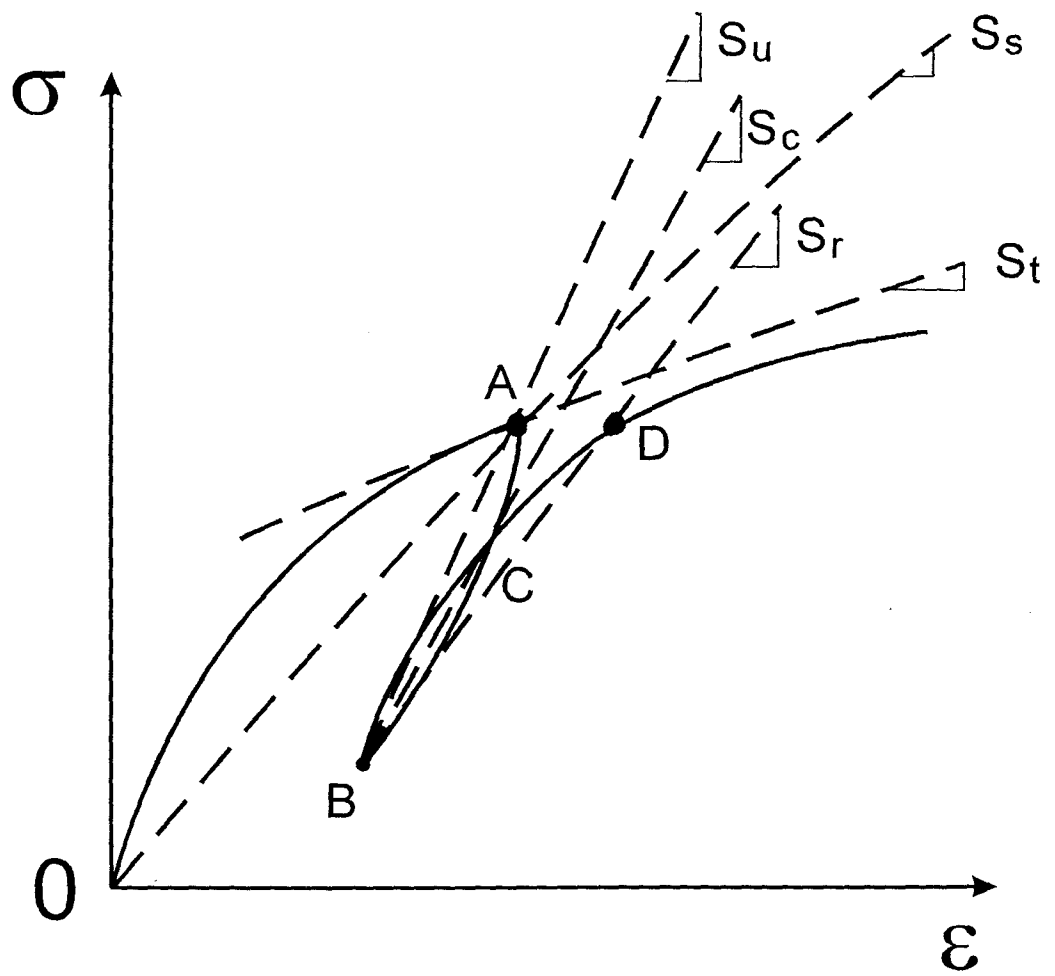
Figure 2. Definition of Soil Modulus

What is the water content? This parameter has a major impact because at low water contents the water binds the particles (especially for fine grained soils) and increases the effective stress between the particles through the suction and tensile skin of water phenomenon. Therefore in this case low water contents lead to high soil moduli. This is why a clay shrinks and becomes very stiff when it dries. At the same time at low water contents the compaction of coarse grain soils is not as efficient as it is at higher water contents because the lubrification effect of water is not there. Therefore in this case low water contents lead to low moduli. As the water content increases, the water occupies more and more room and gets to the point where it pushes the particles apart thereby increasing compressibility and reducing the modulus.

What has the soil been subjected to in the past? This is referred to as the stress history factor. If the soil has been prestressed in the past it is called overconsolidated. This prestressing can come from a glacier which may have been 100 meters thick 10,000 years ago and has now totally melted. This prestressing can also come from the drying and wetting cycles of the seasons in arid parts of the world. If the soil has not been prestressed in the past, in other words if today's stress is the highest stress experienced by the soil and if the soil is at equilibrium under this stress, the soil is normally consolidated. An over-consolidated (OC) soil will generally have higher moduli than the same normally consolidated (NC) soil because the OC soil is on the reload part of the stress strain curve while the NC soil is on the first loading part. Some soils are still in the process of consolidating under their own weight. These are called underconsolidated soils such as the clays deposited offshore the Mississippi Delta where the deposition rate is more rapid than the rate which would allow the pore water pressures induced by deposition to dissipate. These clays have very low moduli.

What about cementation? This refers to the "glue" which can exist at the contacts between particles. As discussed above, low water contents in fine grained soils can generate suction in the water strong enough to simulate a significant "glue effect" between particles. This effect is temporary as an increase in water content will destroy it. Another glue effect is due to the chemical cementation which can develop at the contacts. This cementation can be due to the deposition of calcium at the particle to particle contacts for example. Such cementation leads to a significant increase in modulus.

These are some of the most important factors related to the state of the soil and influencing its modulus. In the following part the factors associated with the loading process are discussed.

LOADING FACTORS

In this section it is assumed that the state factors for the soil considered are fixed. In other words the discussion of each of the factors below can be prefaced by saying "all other factors being equal". Also in this section the secant modulus is used.

What is the mean stress level in the soil? The loading process induces stresses in the soil. These stresses can be shear stresses or normal stresses or a combination of both. At one point and at any given time in a soil mass there is a set of three principal normal stresses. The mean of these three stresses has a significant influence on the soil modulus. This is also called the confinement effect. Figure 1(a) shows an example of two stress strain curves at two different confinement levels. As common sense would indicate, the higher the confinement is the higher the

soil modulus will be. A common model for quantifying the influence of the confinement on the soil modulus is given on Figure 3(a) and is usually attributed to the work of Kondner. According to this model, the modulus is proportional to a power law of the confinement stress. The modulus E_0 is the modulus obtained when the confinement stress is equal to the atmospheric pressure p_a. A common value for the power exponent a in Figure 3(a) is 0.5.

What is the strain level in the soil? The loading process induces strains in the soil mass. Because soils are nonlinear materials, the secant modulus depends on the mean strain level in the zone of influence. In most cases the secant modulus will decrease as the strain level increases because the stress strain curve has a downward curvature. Note that an exception to this downward curvature occurs when the results of a consolidation test is plotted as a stress strain curve on arithmetic scales for both axes. Indeed in this case the stress strain curve exhibits an upward curvature because the increase in confinement brought about by the steel ring is more influential than the decrease in modulus due to the increase in strain in the soil. In the triaxial test, the stress strain curve can be fitted with a hyperbola and the associated model for the modulus is shown on Figure 3(b). This hyperbolic model is usually attributed to the work of Duncan. In this model (Figure 3(b)) E_0 is the initial tangent modulus also equal to the secant modulus for a strain of zero. The parameter s is the asymptotic value of the stress for a strain equal to infinity. In that sense it is related to the strength of the soil.

What is the strain rate in the soil? Soils like many other materials are viscous. This means that the faster a soil is loaded, the stiffer it is and therefore the higher the modulus is. In some instances the reverse behavior is observed. Figure 3(c) shows an example of two stress strain curves obtained by loading the soil at two drastically different strain rates. The strain rate is defined as the strain accumulated per unit of time. The modulus usually varies as a straight line on a log-log plot of modulus versus strain rate. The slope of that line is the exponent b in Figure 3(c). In clays, common values of this exponent vary from 0.02 for stiff clays to 0.1 for very soft clays. In sands common values of b vary from 0.01 to 0.03. The modulus E_0 is the modulus obtained at a reference strain rate. Much of the work on this model has been done at Texas A&M University.

What is the number of cycles experienced by the soil? If the loading process is repeated a number of times, the number of cycles applied will influence the soil modulus. Again referring to the secant modulus, the larger the number of cycles the smaller the modulus becomes. This is consistent with the accumulation of movement with an increasing number of cycles. The model used to describe this phenomenon is shown on Figure 3(d). The exponent c in the model is negative and varies significantly. The most common values are of the order of -0.1 to -0.3. Much of the work on this model has been done at Texas A&M University.

Is there time for the water to drain during the loading process? Two extreme cases can occur: drained or undrained loading. The undrained case may occur if the drainage valve is closed during a laboratory test or if the test is run sufficiently fast in the field. The time required to maintain an undrained behavior or to ensure that complete drainage takes place depends mainly on the soil type. For example a 10 minute test in a highly plastic clay is probably undrained while a 10 minute test in a clean sand is probably a drained test. The Poisson's ratio is sensitive to whether or not drainage takes place. For example if no drainage takes place during loading in a clay it is common to assume a Poisson's ratio equal to 0.5. On the other hand if complete drainage takes place (excess pore pressures are kept equal to zero), then a Poisson's ratio value of 0.35 may be reasonnable. The difference between the two calculated moduli is the difference between the undrained modulus and the drained modulus. Note that the shear modulus remains theoretically
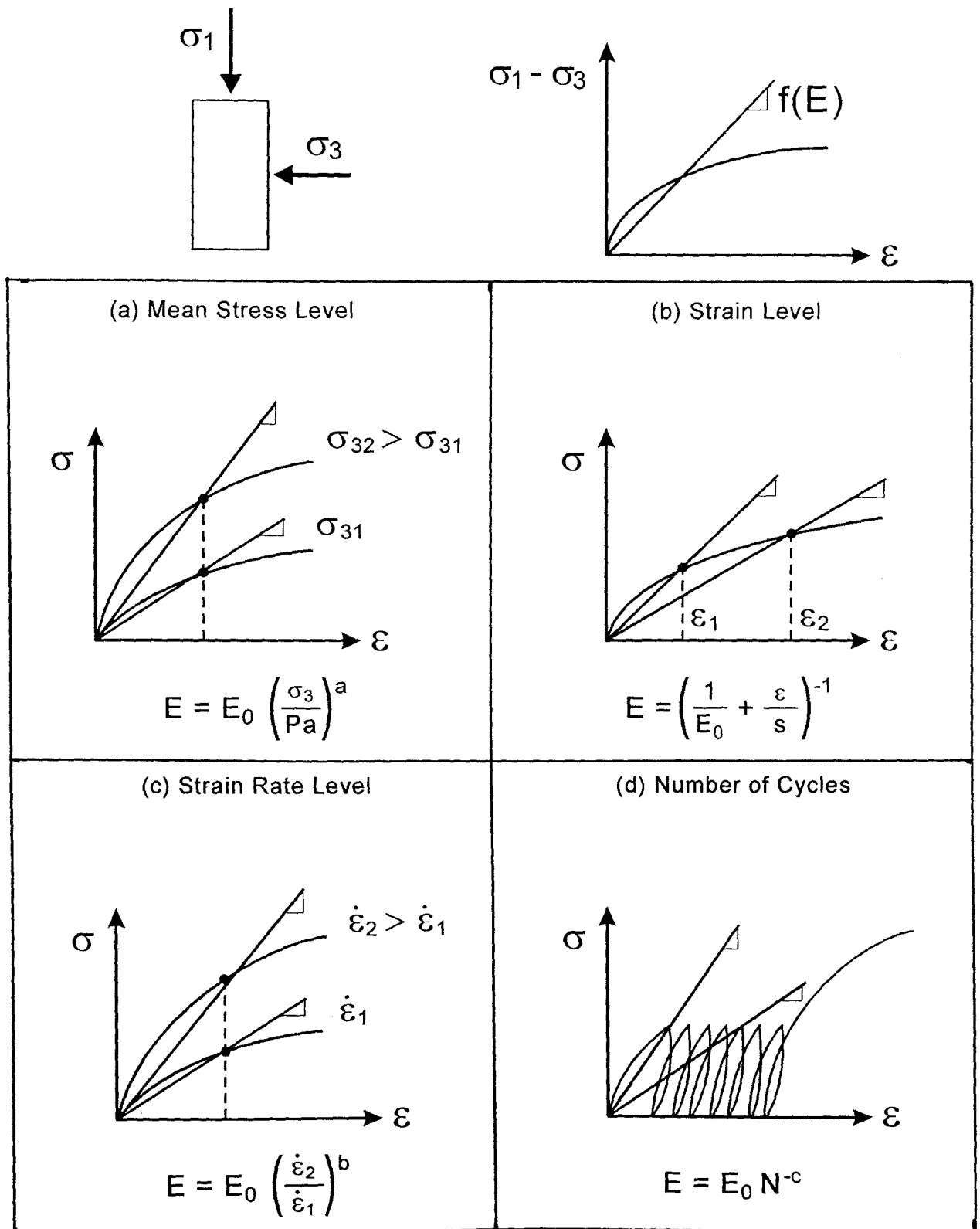
Figure 3. Influencing Factors for Soil Moduli

constant when the drainage varies. Note also that the Poisson's ratio can be larger than 0.5 if the soil dilates during shear associated with compression.

MODULI FOR VARIOUS FIELDS OF APPLICATION

The modulus is useful in many fields of geotechnical engineering. It is clear by now that the modulus required for one field may be significantly different from a modulus for another field.

In the case of shallow foundations, the mean stress level applied under the foundation is often between 100 and 200 kPa. The normal strain level in the vertical direction is about 0.01 or less and is typically associated with a movement of about 25 mm. The rate of loading is extremely slow because that strain occurs first at the construction rate and is then sustained over many years. The number of cycles is one unless cycles due to daily temperatures or other cyclic loading (such as compressor foundations) are included. Example values of the modulus in this case are 10,000 to 20,000 kPa.

In the case of deep foundations, the mean stress level varies because the side friction on the piles occurs over a range of depth, while the point resistance occurs at a relatively large depth. The strain level at the pile point is usually smaller than in shallow foundations because a percentage of the load dissipates in friction before getting to the pile point. The strain rate is similar to the case of shallow foundations with rates associated with months of construction and years of sustained loads. High strain rates do occur however in the case of earthquake or wave loading. Cycles can be a major issue for earthquake loading of buildings and bridges or for wave loading of offshore structures. Because deep foundations are used in very different types of soils and for very different types of loading, the moduli vary over a much wider range of values than for shallow foundations.

In the case of slope stability and retaining structures, movements are associated with the deformation of the soil mass essentially under its own weight. Therefore the stress level corresponds to gravity induced stresses. The strains are usually very small and the strain rate is again associated with the rate of construction at first and the long term deformation rate during the life of the slope or of the retaining structure. Cycles may occur due to earthquakes or other cyclic phenomenons. For properly designed slopes and retaining structures, the moduli tend to be higher than in foundation engineering because the strain levels tend to be smaller.

In the case of pavements, the mean stress level in the subgrade is relatively low. The pressure applied to the pavement is of the order of 200 kPa for car tires, 500 kPa for truck tires, and 1700 kPa for airplane tires. However, the vertical stress at the top of the subgrade under a properly designed pavement may be only one tenth of the tire pressure applied at the surface of the pavement. The strain level is very low because the purpose of the pavement is to limit long term deformations to movements measured in millimeters if not in tenths of millimeters. Typical strain levels are 0.001 or less at the top of the subgrade. The rate of loading is very high and associated with the passing of a traveling vehicle. The loading time is of the order of milliseconds for a car at 100 km/h but is measured in hours for an airplane parked at the gate. The number of cycles is tied to the number of vehicles traveling on the pavement during the life of the pavement. This number varies drastically from less than a million of vehicle cycles for small roads to tens of millions for busy interstates. Typical modulus values range from 20,000 kPa to 150,000 kPa.

MODULUS OR STIFFNESS?

The modulus E has been defined in Figure 1. It has units of force per unit area (kN/m^2). The stiffness K is defined here as the ratio of the force applied on a boundary through a loading area divided by the displacement experienced by the loaded area. It has units of force per unit length (kN/m). The loaded area is typically a plate which can be square or circular or in the shape of a ring. There is a relationship between the modulus and the stiffness. For the case of a circular plate having a diameter B, the relationship is of the form:

$$E = f(K/B)$$

This relationship shows that, if the modulus is a soil property, **the stiffness is not a soil property and depends on the size of the loaded area**. Therefore, for an elastic material, the stiffness measured with one test will be different from the stiffness measured with another test if the loading areas are different. Yet, for the same elastic material, the modulus obtained from both tests would be the same. In that sense the use of the stiffness is not as convenient as the modulus and the use of the modulus is preferred.

MODULUS OR COEFFICIENT OF SUBGRADE REACTION?

The modulus E has been defined in Figure 1. It has units of force per unit area (kN/m^2). The coefficient of subgrade reaction k is defined here as the ratio of the pressure applied to the boundary through a loading area divided by the displacement experienced by the loaded area. It has units of force per unit volume (kN/m^3). The loaded area can be a footing (coefficient of vertical subgrade reaction) or a horizontally loaded pile (coefficient of horizontal subgrade reaction). There is a relationship between the modulus and the coefficient of subgrade reaction. For the footing and the pile cases mentioned, that relationship is of the form:

$$E = g(k \times B)$$

where B is the footing width or the pile width or diameter. This relationship shows that, if the modulus is a soil property, **the coefficient of subgrade reaction is not a soil property and depends on the size of the loaded area**. Therefore, if a coefficient of subgrade reaction k is derived from load tests on a footing or a pile of a certain dimension, the value of k cannot be used directly for other footing or pile sizes. Indeed in this case, careful considerations of size and scale must be addressed. The modulus is not affected by this problem. In that sense the use of the coefficient of subgrade reaction is not as convenient as the modulus and the use of the modulus is preferred.