Report
Estimating Dry Density
From
Soil Stiffness & Moisture Content

29 June, 1999

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Problem

If any new method of evaluating soil compaction is to be widely accepted, a firm relationship must be established between this method and the most accepted current methods, measurements of dry density.

Objective

Develop an analytical-empirical relationship between soil stiffness and density. Validate the relationship with data from Humboldt GeoGauge™ measurements and accepted methods of measuring density.

Approach

Began with the analytical-empirical relationship that was developed by BBN Technologies of Cambridge, MA some 4 years ago from the work of Hryciw & Thomann [1].

$$\rho_D = \frac{\rho_0}{1 + 1.2\left[\frac{C}{K} - .3\right]^{.5}} \qquad 1)$$

where

$$C = \frac{(C_1 \sigma_1^P)4a}{(1-\upsilon)}$$

C_1 = is a function of moisture and soil type

σ_1 = is the overburden stress

P = is typically between 1/2 and 1/4

a = is the foot radius

υ = is Poisson's ratio

ρ_D = is the dry density

ρ_0 = is the ideal, void free density

K = is stiffness

Define C for a geographical region or group of soil classes, independent of everything but moisture. Do this based on companion stiffness, moisture content and density measurements. Then use C, measured stiffness and measured moisture content to estimate dry density. Compare the estimates to density measurements made with a nuclear gauge and sand cone.

[1] Roman D. Hryciw & Thomas G. Thomann, "Stress-History-Based Model for Cohesionless Soils", *Journal of Geotechnical Engineering*, Vol. 119, No, 7, July, 1993

Results

Analytical-Empirical Relationship

Early attempts at following this approach revealed two things.

- A more precise estimation was possible when moisture content was broken out of the constant C and
- More precision was possible when the values of C were calculated from a linear relationship with stiffness and moisture content.

Solving equation 1) for C yields

$$C = K\left\{\left[(\rho_0/\rho_D-1)/1.2\right]^2 + 0.3\right\} \qquad 2)$$

If we let C = Cm, where m = (% moisture content by weight)/100), then C can be represented as

$$C = (K/m)\left\{\left[(\rho_0/\rho_D-1)/1.2\right]^2 + 0.3\right\} \qquad 3)$$

This representation allows for moisture content to be included in each estimate of dry density. It also allows the values of C determined from the companion measurements to be fitted to a linear equation with our two independent variables, K and m.

$$C = n(K/m^{.25}) + b \qquad 4)$$

where
n is the slope
and
b is the intercept.

This linear relationship between C, K and m allows a more appropriate value of C to used in the estimate of each dry density as opposed to selecting a limited number of Cs to used over several moisture ranges. Breaking m out of C and using this linear relationship provided closer agreement between measured and estimated dry density in 23% of the cases compared to not doing either.

Numerous other modifications of equation 1) were numerically analyzed relative to actual companion measurement data. The analytical-empirical relationship represented by equations 3 and 4 fit the data the best. Figure 1 is a 3D surface plot of K, M and ρ_D as described by this relationship. The relationship appears to be well behaved in the ranges of density, stiffness and moisture content that most applications will encounter.

Based on the usage of current methods for evaluating compaction and a consensus of GeoGauge™ customers, the following criteria were established for the evaluation of the above approach.

a) Estimates of dry density should be within 5% of the measured values about 70% of the time & within 10% > 90% of the time.
b) The span of measured & estimated densities should be almost the same.
c) A one-to-one correspondence of measured to estimated densities should yield a correlation coefficient of > .3 (typically > .5).

Validation of the Relationship

Five hundred and seventy seven (577) companion measurements were made in California, Ohio, Florida, Missouri, New York, North Carolina and Virginia by the FHWA, California Polytechnic Institute, the H. C. Nutting Co., the City of San Jose, the FDOT, the MODOT, the NYSDOT and the NCDOT. These measurements were made largely independent of Humboldt. The data, the estimates of dry density and the comparisons of the estimates to direct measurements are presented in Appendices 1 through 9.

Each appendix contains the following information.

- Multiple plots of raw data; density vs. stiffness vs. moisture content
- Summaries of how well C was determined form a function of stiffness & moisture segregated by groups of similarly performing soils
- Plots of estimated vs. measured density in terms of percentage difference and one-to-one correspondence
- All the data used to determine C and numerical data for all density estimates, segregated by data that was used to determine C and data that was not[2]

It was evident that several classes or groups of similarly performing soils were represented by the data from each source. In some cases, when C was plotted against a function of K and m, the presence of more than one linear relationship was apparent. In other cases, there was a clustering of values of C that were calculated from companion measurements. When one or both of these conditions coincided with test sites or locations, the data was correspondingly segregated and analyzed independently. This greatly improved the results of the analysis in meeting the criteria stated earlier. Since only the California Polytechnic Institute provided soil classifications with its data, the validity of this operation will need to be confirmed with the sources of the data.

It is also evident that the relationship represented by equations 3 and 4 will not provide satisfactory estimates of density for every soil. Soils due to stabilization additives, construction methods, site conditions or just their nature are apparently atypical. The data from the FDOT is a good example. As can be seen from the raw data in Appendices 5 and 6, that sandy, limestone stabilized soil are not typical of the soil behavior illustrated in the other appendices. For such soils, it was found that by using the relationship represented by equations 1 and 4 satisfactory estimates of density were possible. Figure 2 is a 3D surface plot of K, M and ρ_D as described by equations 1 and 4.

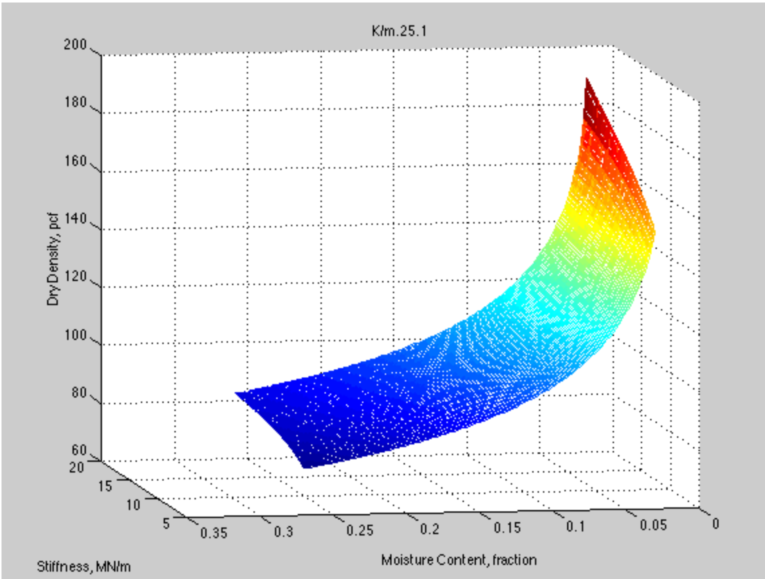
[2] Due to the volume of data, this information is omitted from the pdf version of the report. A hard copy of this information is available upon request.

Table 1 summaries the results presented in the appendices. The 3 evaluation criteria applied across the 10 data sources are met 96% of the time. Only criteria a) is missed in the MODOT data.

Conclusions

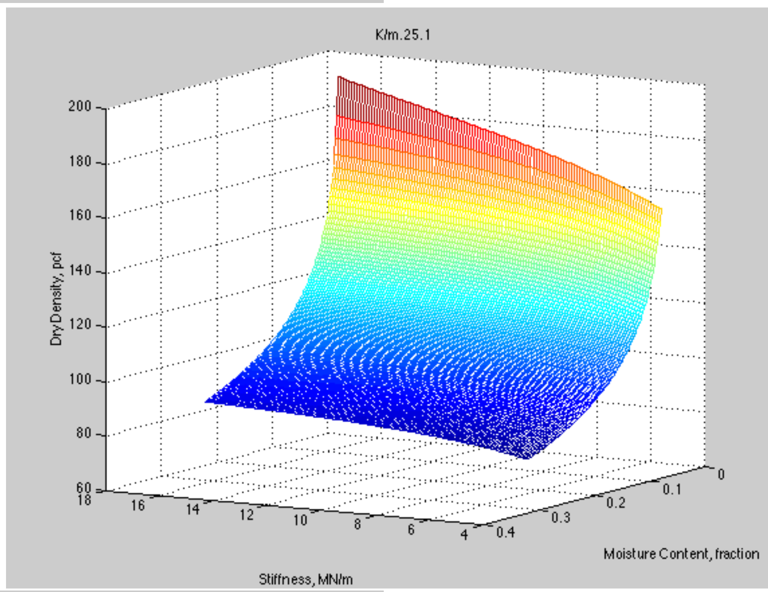
An analytical-empirical relationship has been developed that allows the estimation of dry density from soil stiffness and moisture content within tolerances that are typical in the use current field measurements. The successful application of this relationship requires that it be adjusted for groups of similarly performing soils and atypical soils. This relationship firmly connects soil stiffness, as measured by the Humboldt GeoGauge™, with dry density. This relationship in conjunction with companion measurements of moisture content and stiffness is a potential alternative method for determining dry density.

Figure 1:
Surface Plot of K, M and ρ_D
as Described by Equations 3) & 4)



$$\rho_D = \frac{\rho_0}{1 + 1.2\left[\frac{mC}{K} - .3\right]^{.5}}$$

$$C = n(K/m^{.25}) + b$$



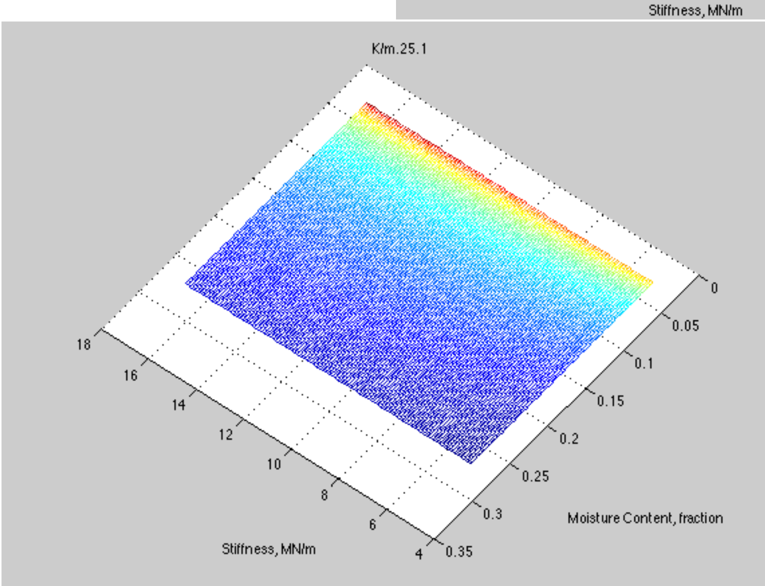
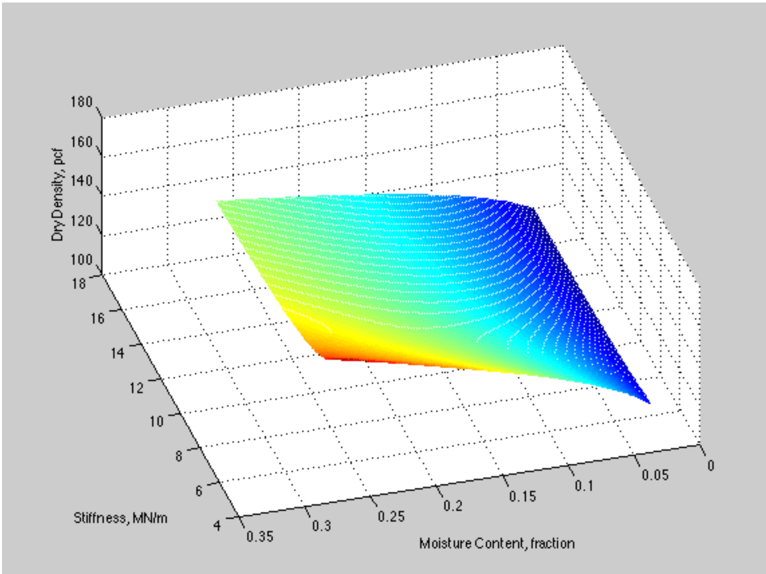
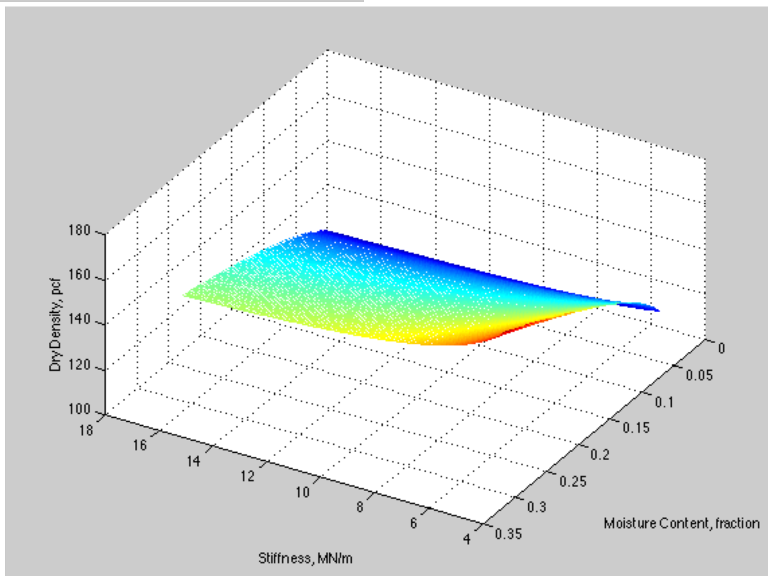


Figure 2:
Surface Plot of K, M and ρ_D
as Described by Equations 1) & 4)



$$\rho_D = \frac{\rho_0}{1 + 1.2\left[\frac{C}{K} - .3\right]^{.5}}$$

$$C = n(K/m^{.25}) + b$$



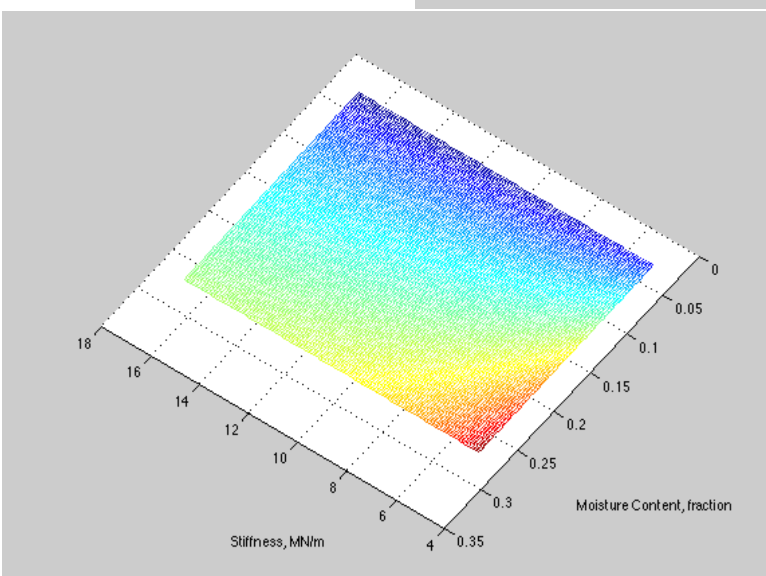
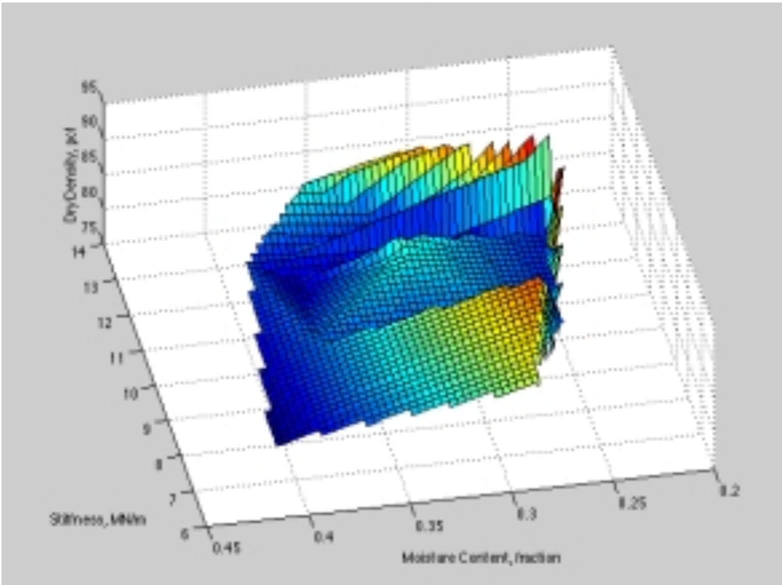


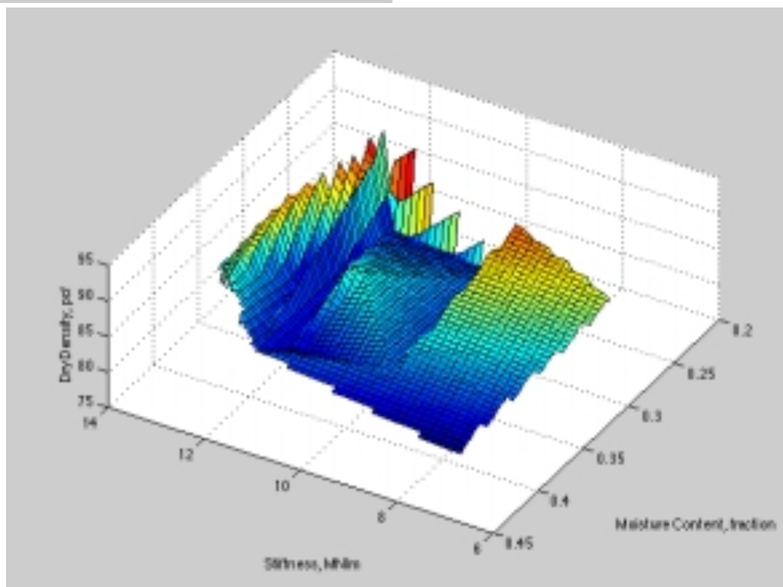
Table 1: Summary of Results

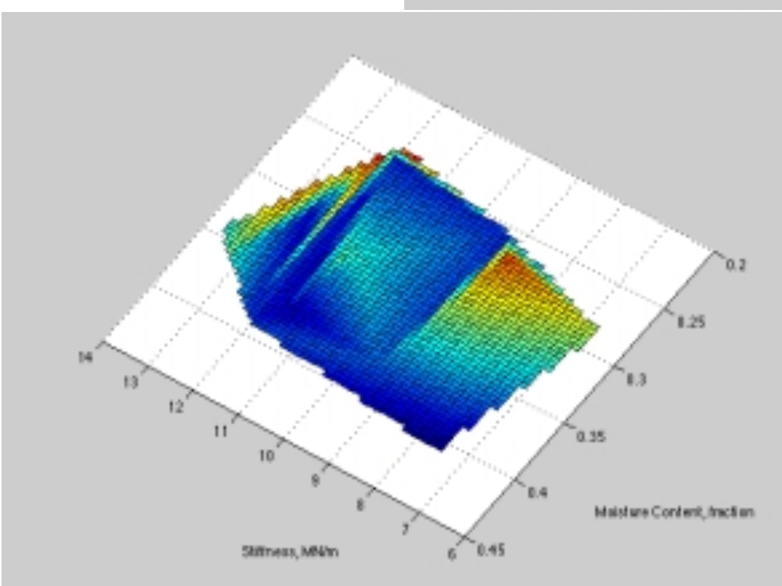
Data Source	**Number of Companion Measurements**	**Relationship Used**	Δ%, ρ_D (GeoGauge) re ρ_D (Nuc) (percentage of estimates within 5, 10 and 15 % of the direct measurements)			**Density Span GeoGauge/Nuc (pcf)**	**R^2** (correlation coefficient) **ρ_D (Nuc) vs. ρ_D (GeoGauge)**
			5%	**10%**	**15%**		
Cal. Poly.	80	Eq. 3 & 4	82%	100%	-	32/35	0.83
H.C. Nutting	66	Eq. 3 & 4	95%	5%	-	34/33	0.86
San Jose	120	Eq. 3 & 4	70%	99%	100%	33/27	0.33
FDOT (field)	112	Eq. 1 & 4	88%	100%	-	23/18	0.43
FDOT (lab)	34	Eq. 1 & 4	97%	100%	-	10/9	0.39
MODOT	30	Eq. 3 & 4	60%	100%	-	39/36	0.77
NYSDOT	50	Eq. 3 & 4	90%	100%	-	0.31/0.34 Mg/m^3	0.51
NCDOT	17	Eq. 3 & 4	100%	-	-	17/16	0.90
FHWA	60	Eq. 3 & 4	88%	100%	-	66/62	0.94

Appendix 1
Analysis of NCDOT Data

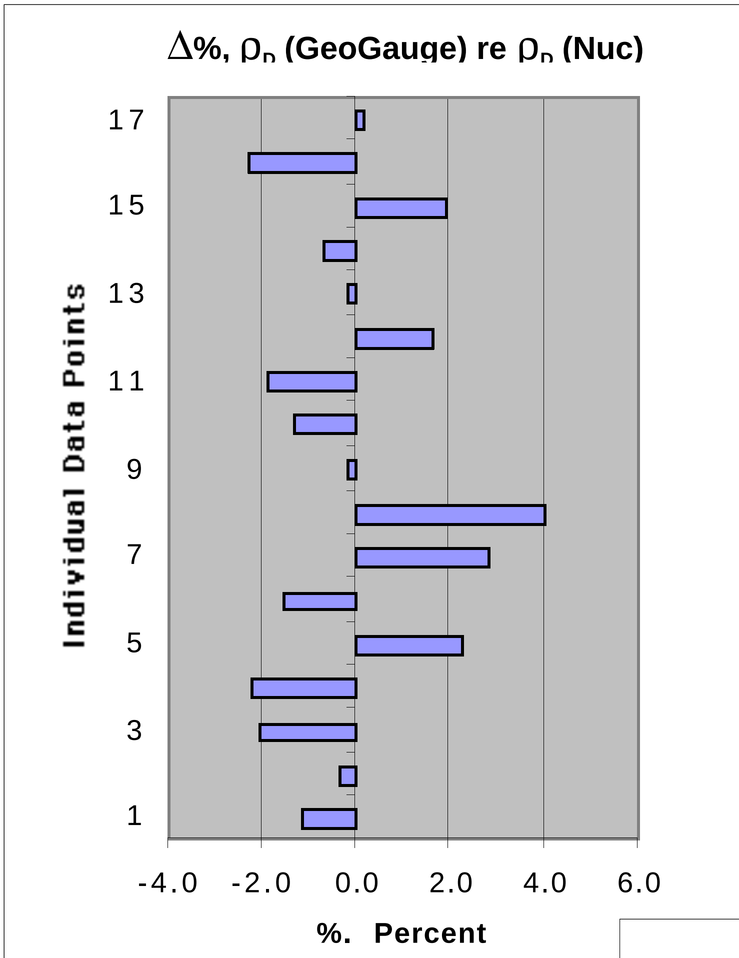
NCDOT Raw Data







NCDOT
Data Analysis Summary



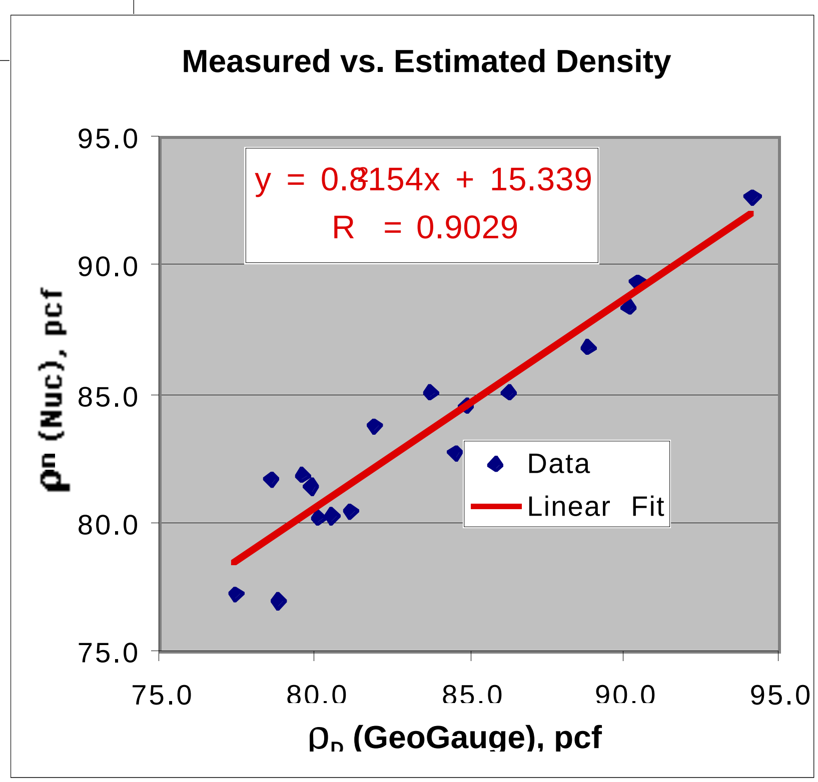
$$\rho_D = \frac{\rho_0}{1 + 1.2\left[\frac{mC}{K} - .3\right]^{.5}}$$

$$C = n(K/m^{.25}) + b$$

Determination of C

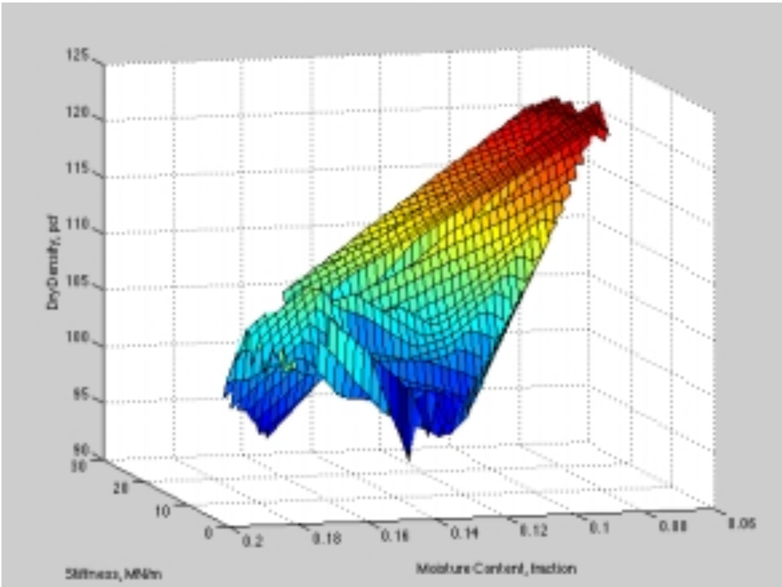
Soil Group #1:
- $C = 3.7095(K/m^{.25}) + 8.2414$
- $R^2 = 0.8987$

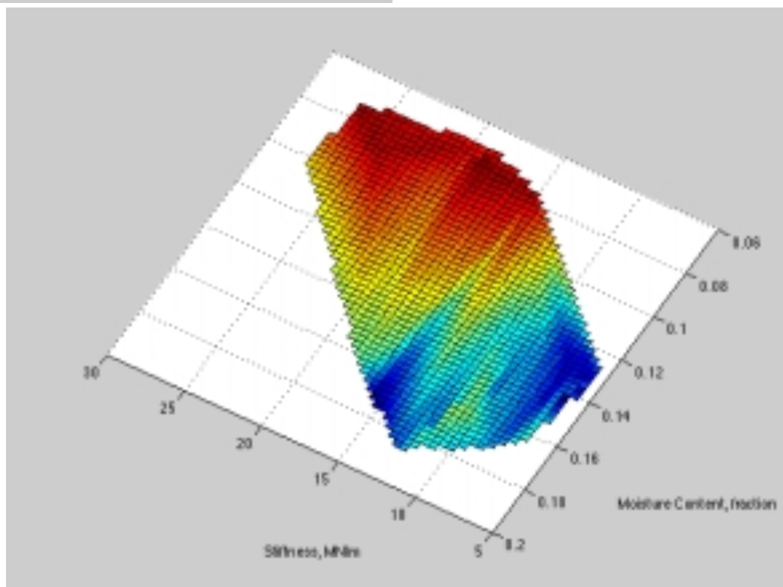
Soil Group #2:
- $C = 5.6425(K/m^{.25}) + 3.4313$
- $R^2 = 0.8331$

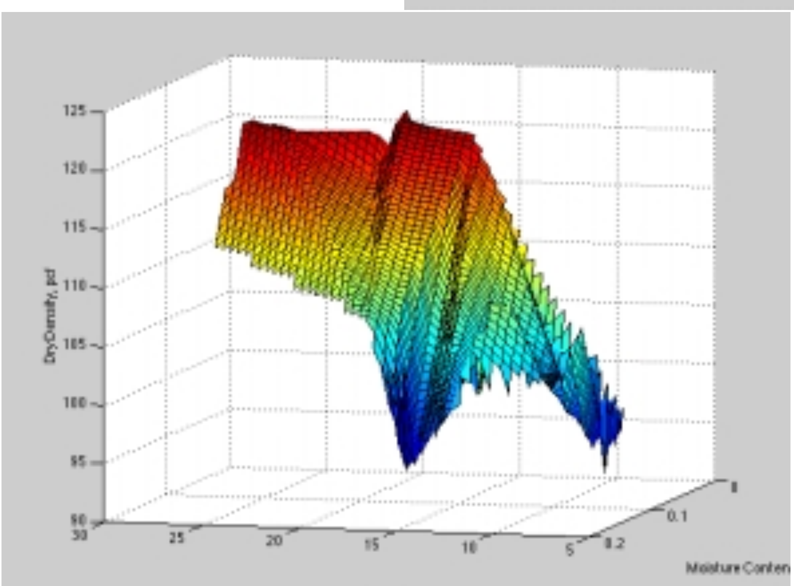


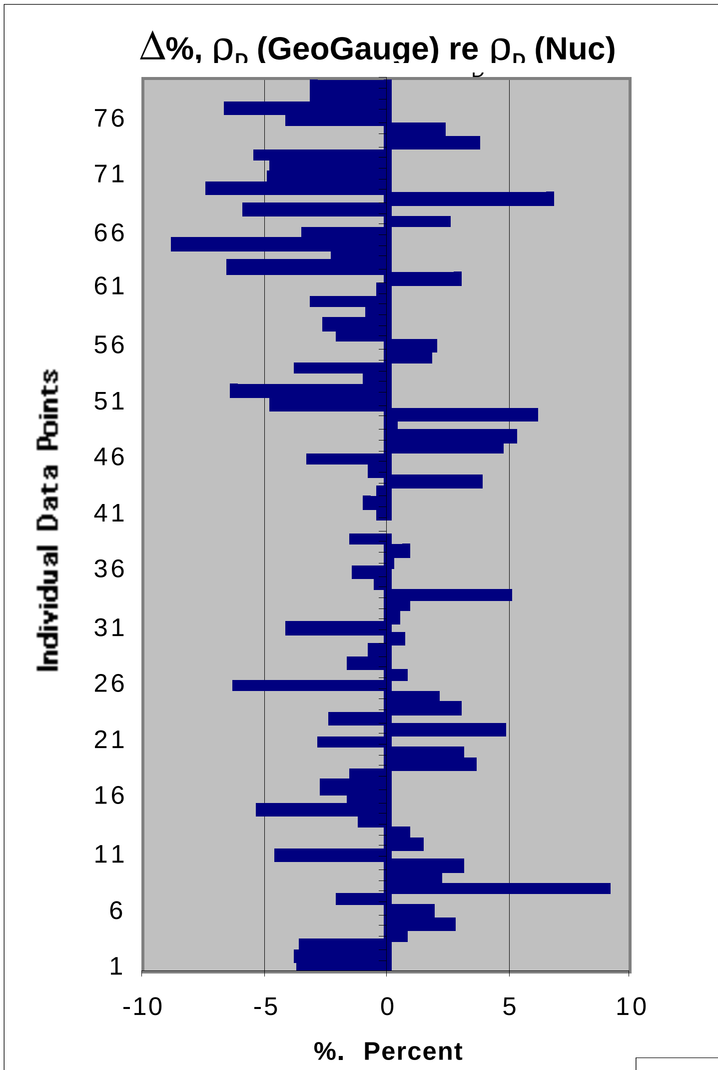
Appendix 2
Analysis of Cal. Poly. Data

Cal. Poly. Raw Data









$$\rho_D = \frac{\rho_0}{1 + 1.2\left[\frac{mC}{K} - .3\right]^{.5}}$$

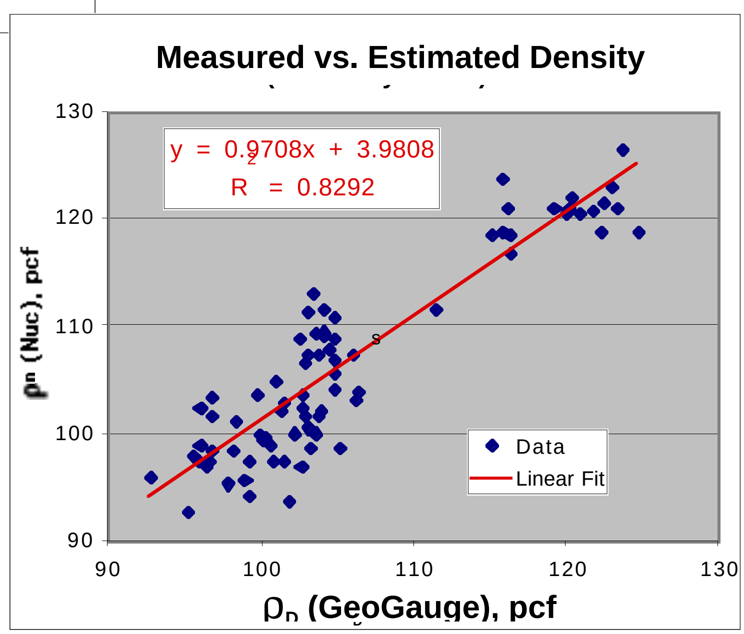
$$C = n(K/m^{.25}) + b$$

Determination of C

Site #1, AASHTO A-2-6(U):
$C = 4.4561(K/m^{.25}) + 12.704$
$R^2 = 0.8943$

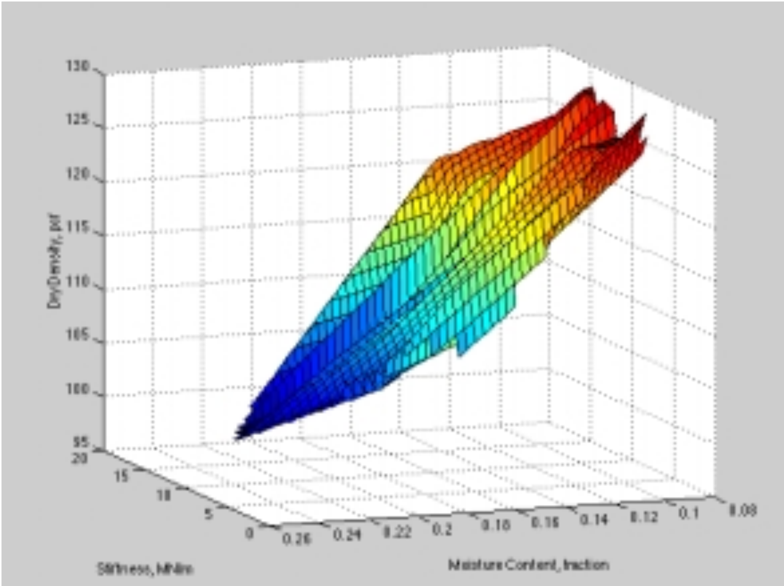
Site #2, AASHTO A-6(6):
$C = 3.765(K/m^{.25}) + 19.165$
$R^2 = 0.8378$

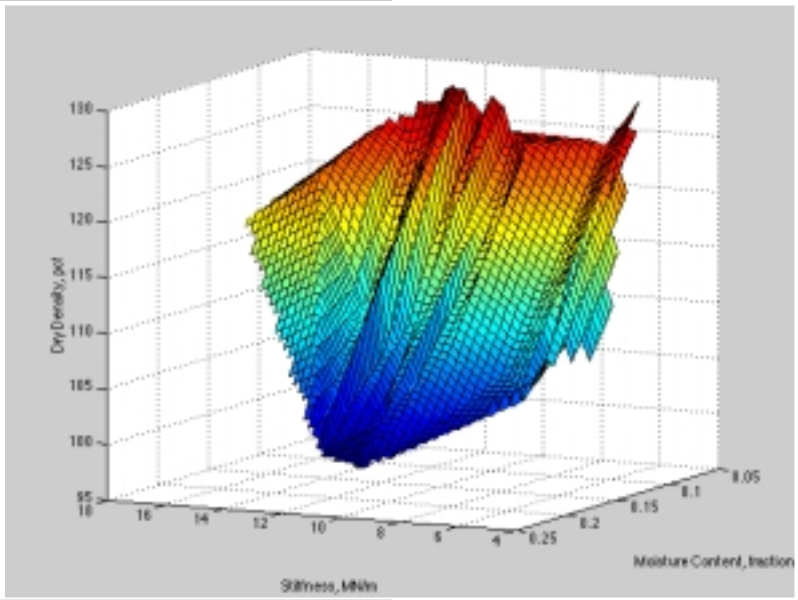
Site #3, AASHTO A-6(7):
$C = 4.2431(K/m^{.25}) – 4.8947$
$R^2 = 0.8199$

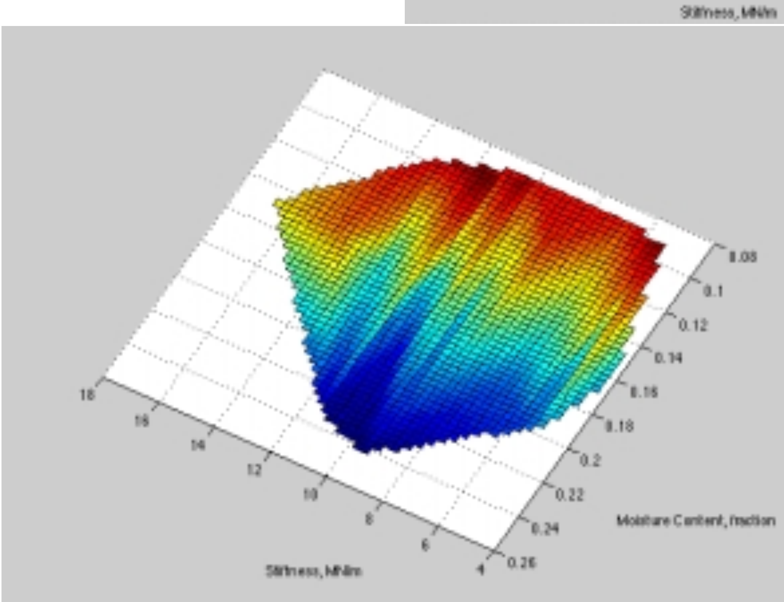


Appendix 3
Analysis of H. C. Nutting Data

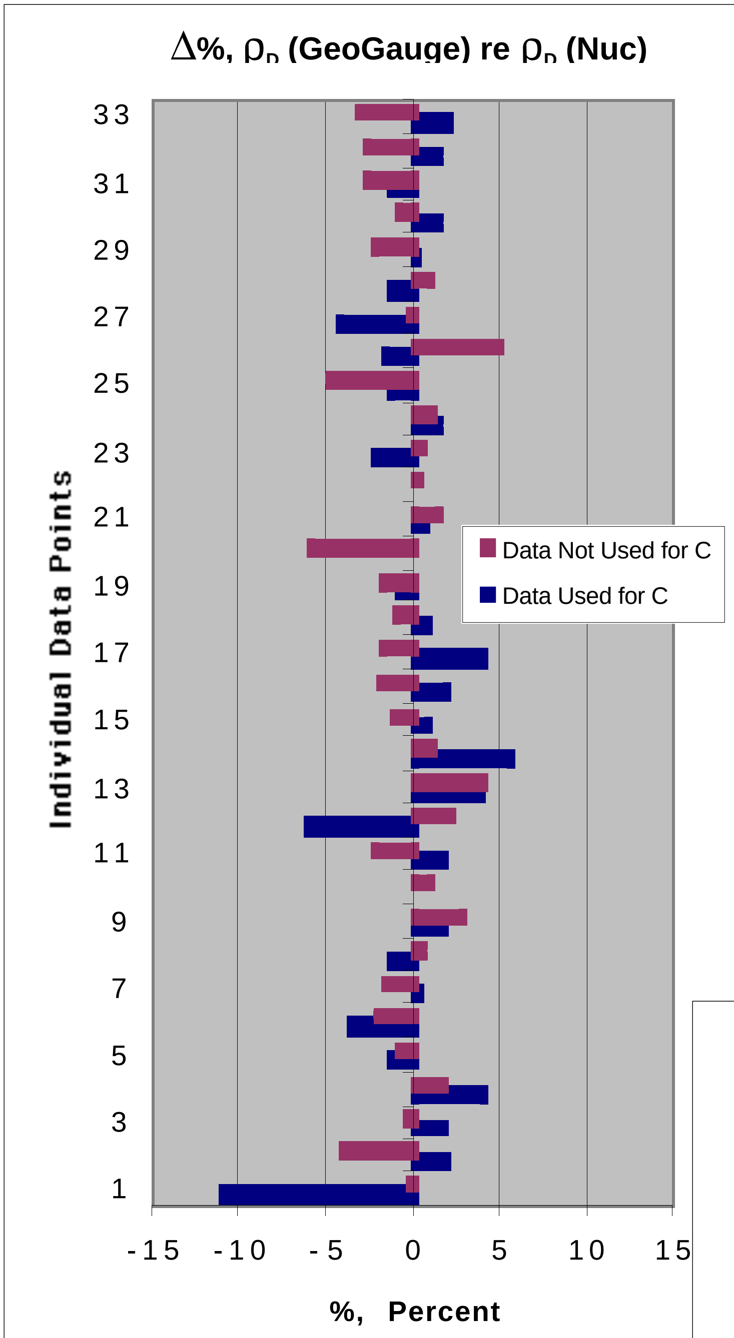
H. C. Nutting Raw Data





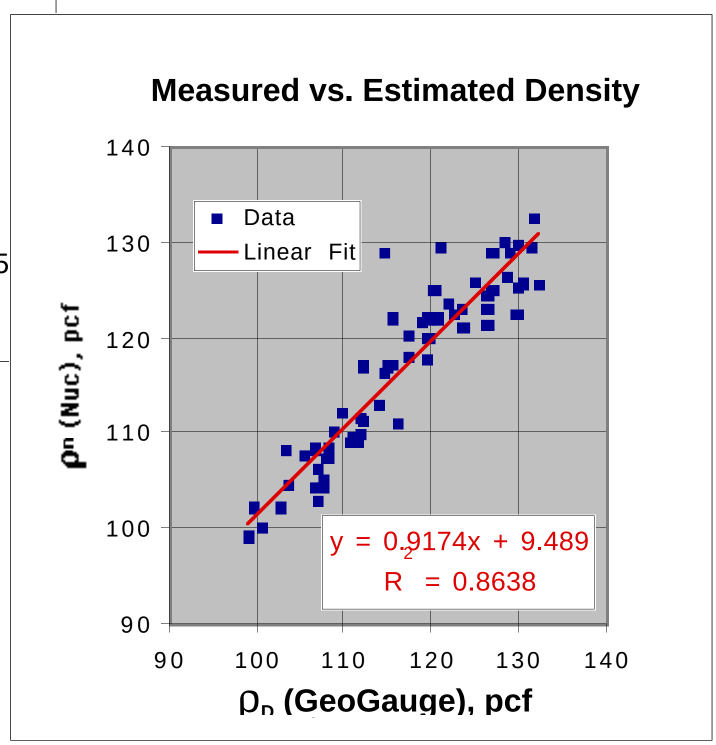


H. C. Nutting Co., Cincinnati, OH
Data Analysis Summary



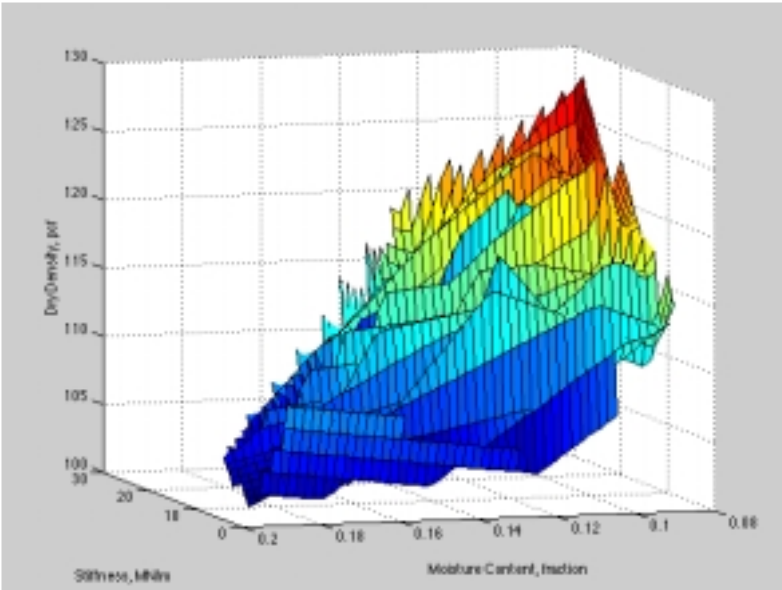
Determination of C

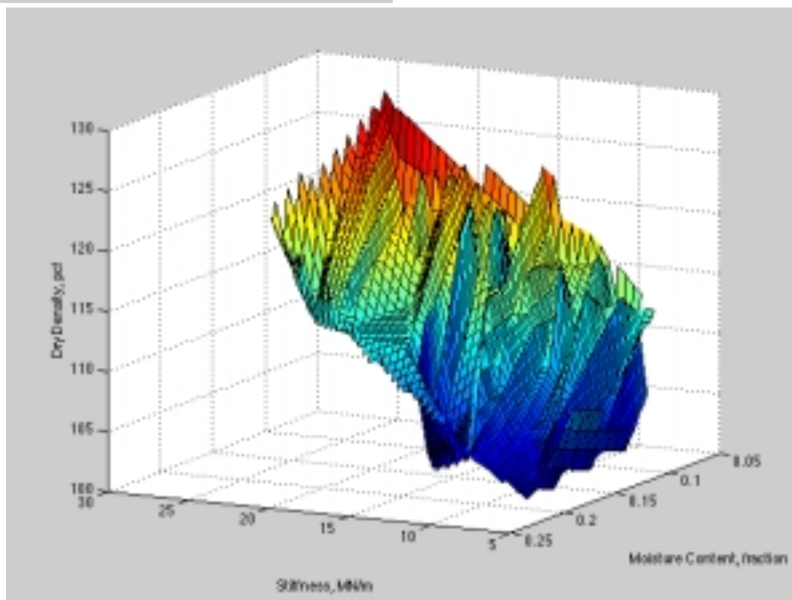
Soil Group #1:
$C = 2.8335(K/m^{.25}) + 11.465$
$R^2 = 0.7417$
Soil Group #2:
$C = 3.1484(K/m^{.25}) + 2.6727$
$R^2 = 0.9693$
Soil Group #3:
$C = 2.8146(K/m^{.25}) + 10.44$
$R^2 = 0.9414$

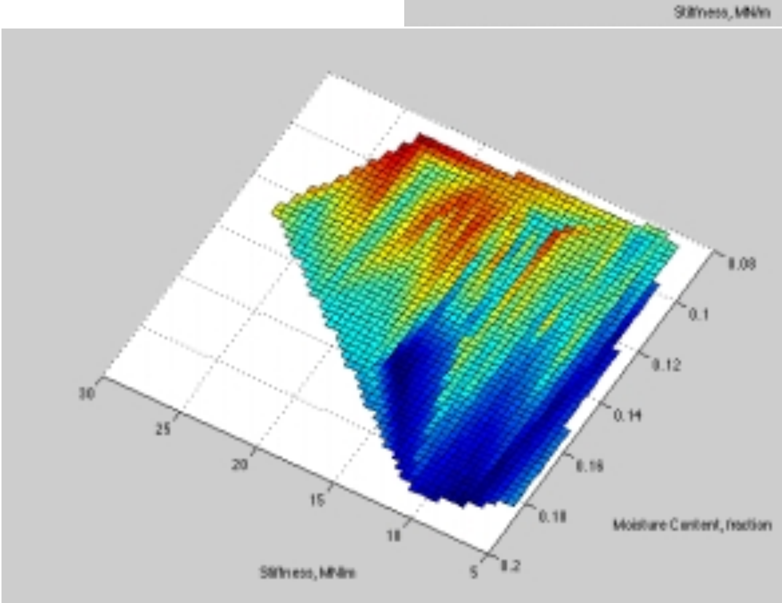


Appendix 4
Analysis of San Jose Data

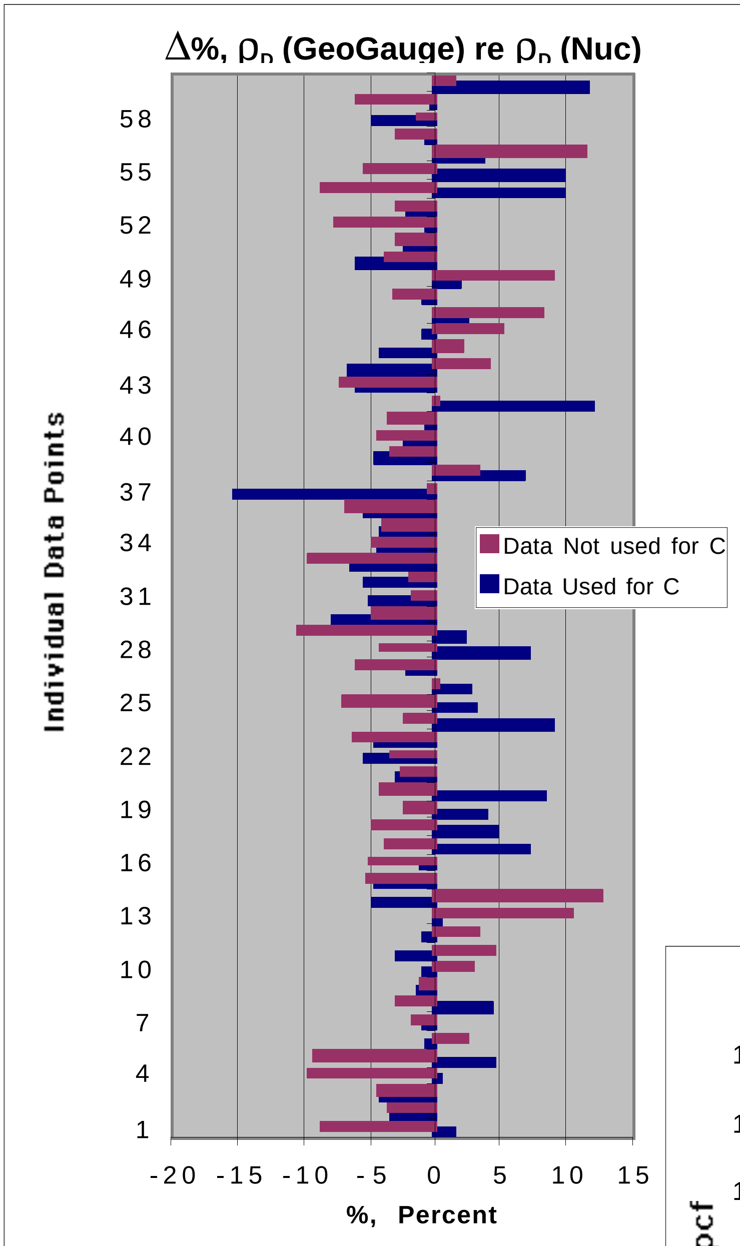
San Jose Raw Data





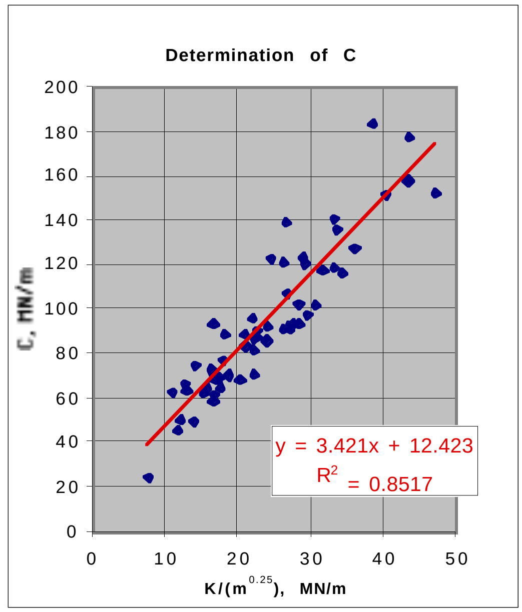


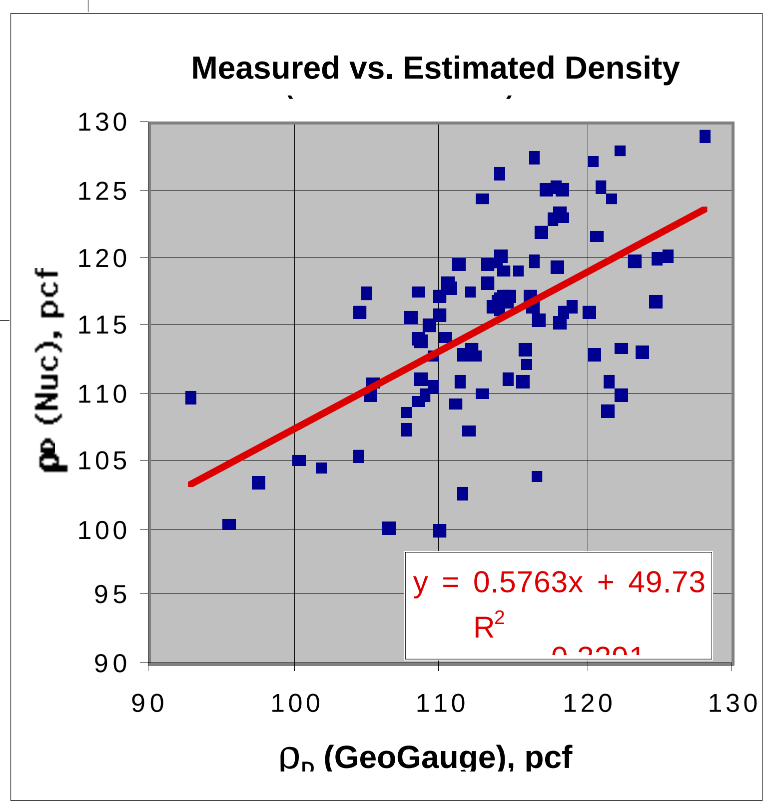
City of San Jose, CA
Data Analysis Summary



$$\rho_D = \frac{\rho_0}{1 + 1.2\left[\frac{mC}{K} - .3\right]^{.5}}$$

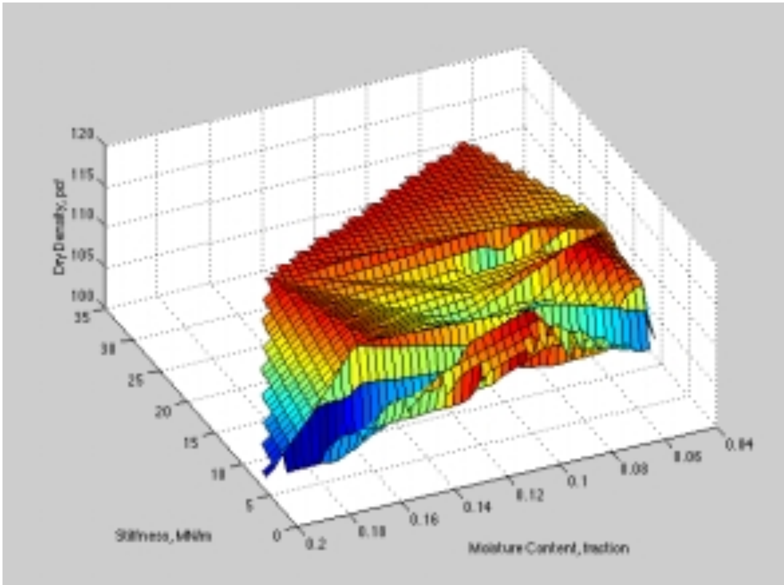
$$C = n(K/m^{.25}) + b$$

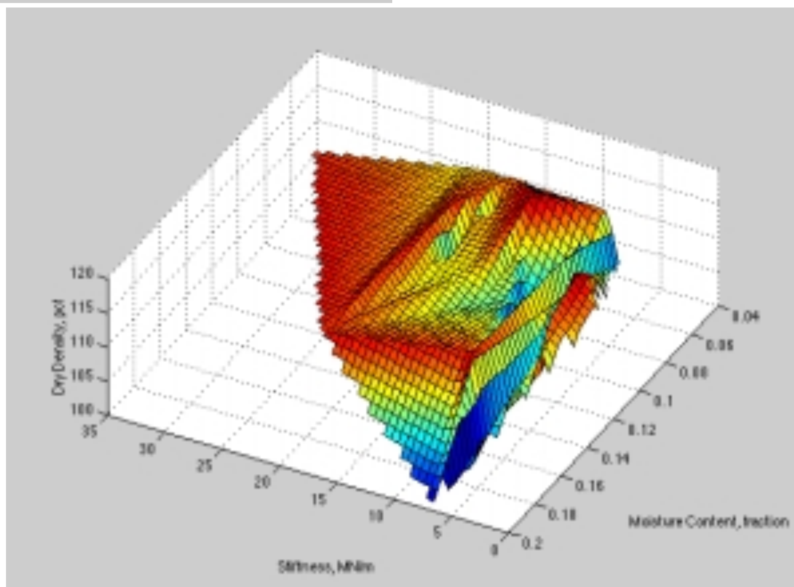


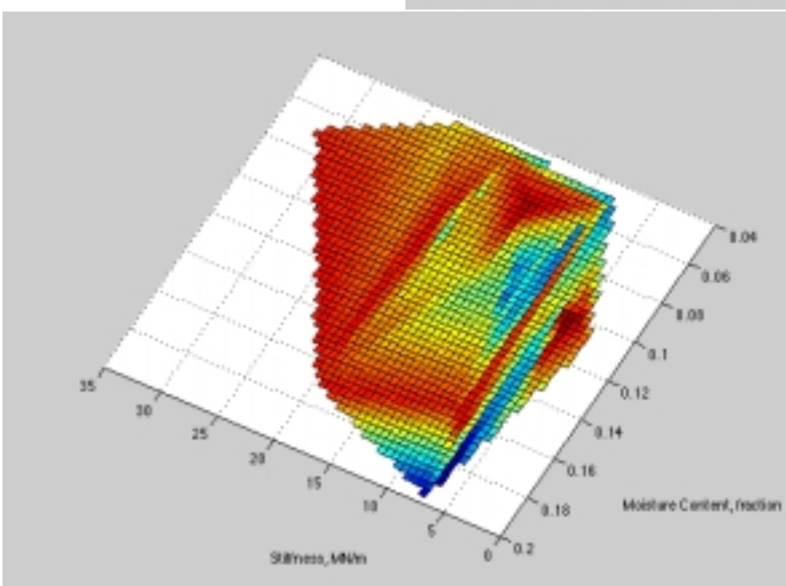


<u>Appendix 5</u>
Analysis of FDOT Field Data

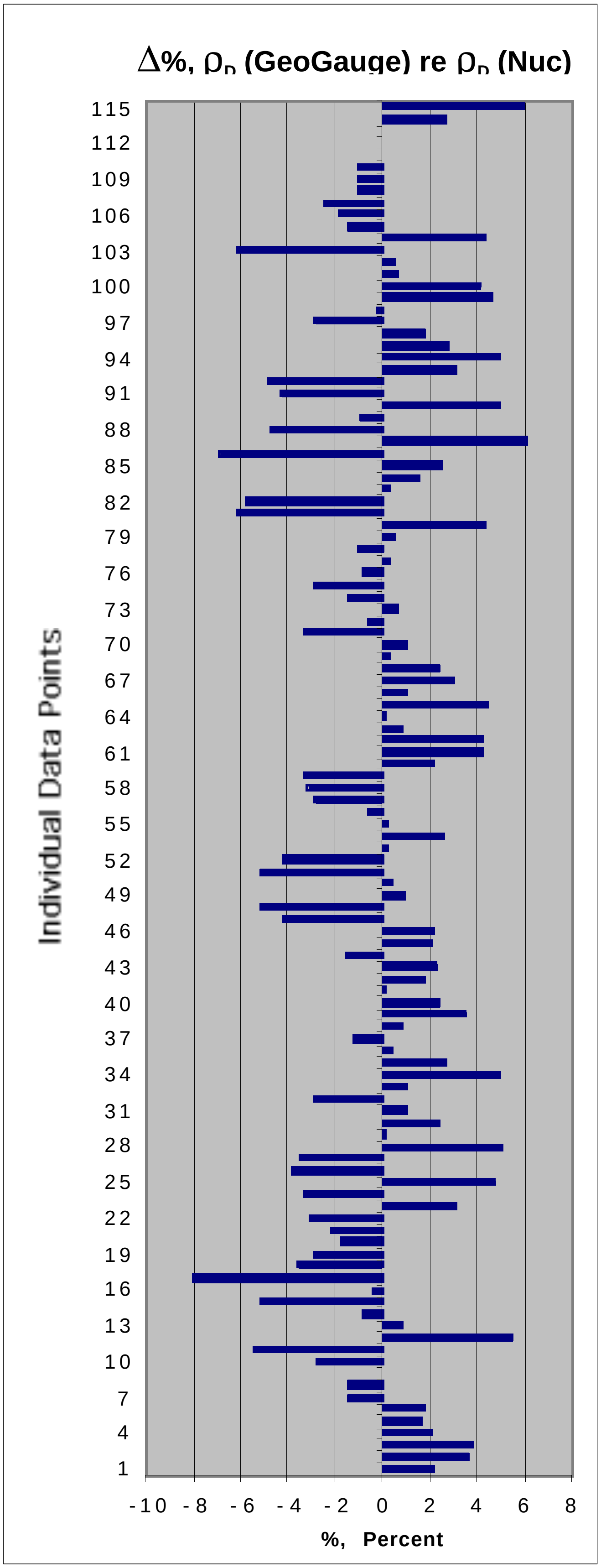
FDOT Raw Field Data







FDOT
Field Data Analysis Summary



$$\rho_D = \frac{\rho_0}{1 + 1.2\left[\frac{C}{K} - .3\right]^{.5}}$$

$$C = n(K/m^{.25}) + b$$

Determination of C

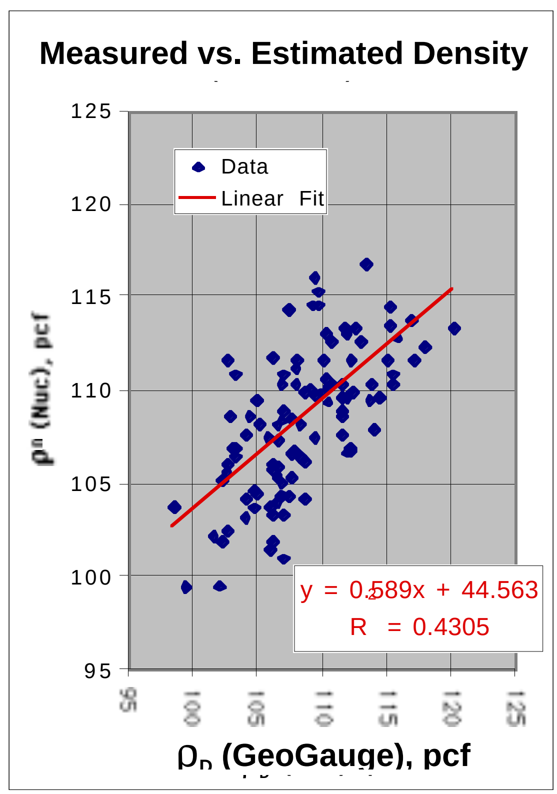
Soil Group #1:
$C = 0.3536(K/m^{.25}) + 1.8587$
$R^2 = 0.9439$

Soil Group #1a:
$C = 0.4613(K/m^{.25}) + 1.0223$
$R^2 = 0.97$

Soil Group #2:
$C = 0.5391(K/m^{.25}) + 0.1964$
$R^2 = 0.9568$

Soil Group #3:
$C = 0.4126(K/m^{.25}) + 0.8955$
$R^2 = 0.9828$

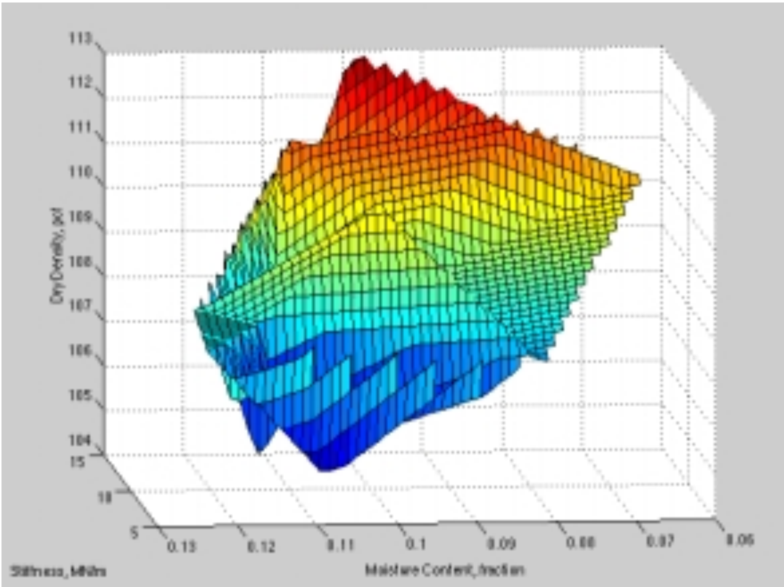
Soil Group #4:
$C = 0.1288(K/m^{.25}) + 6.48$
$R^2 = 1$

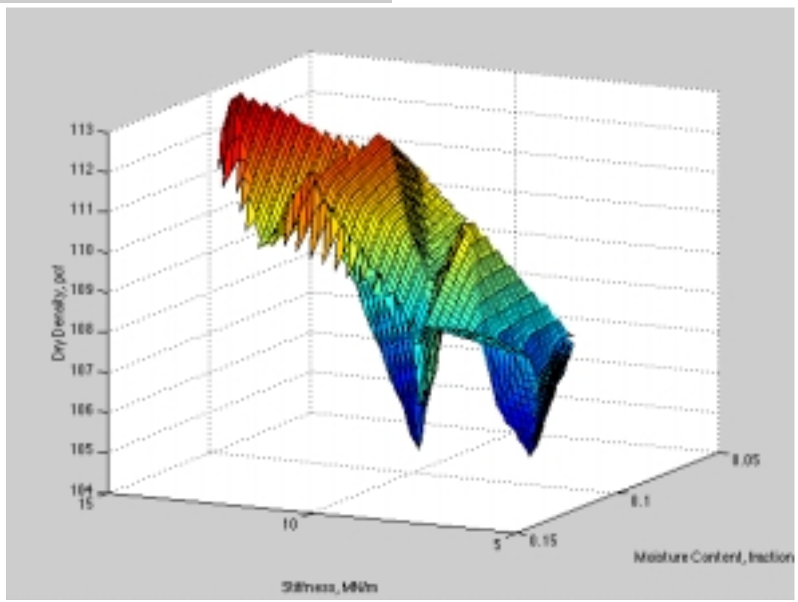


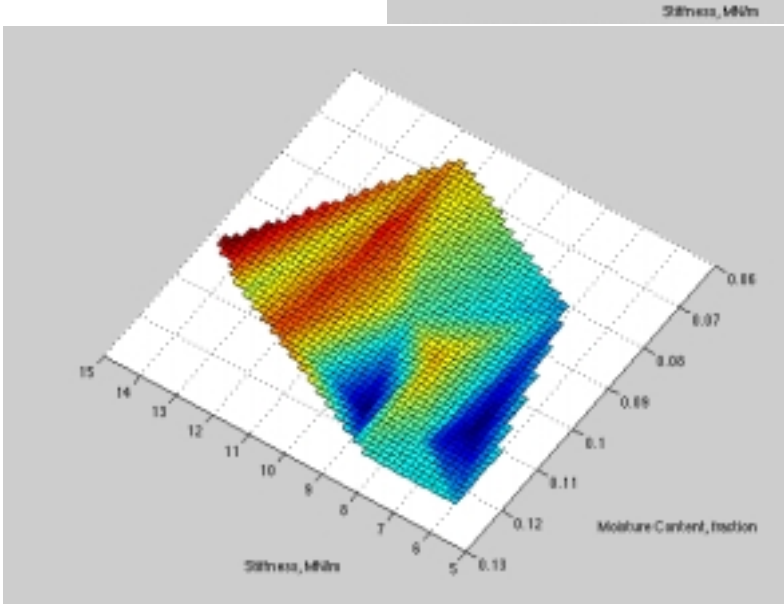
Appendix 6

Analysis of FDOT Lab Data

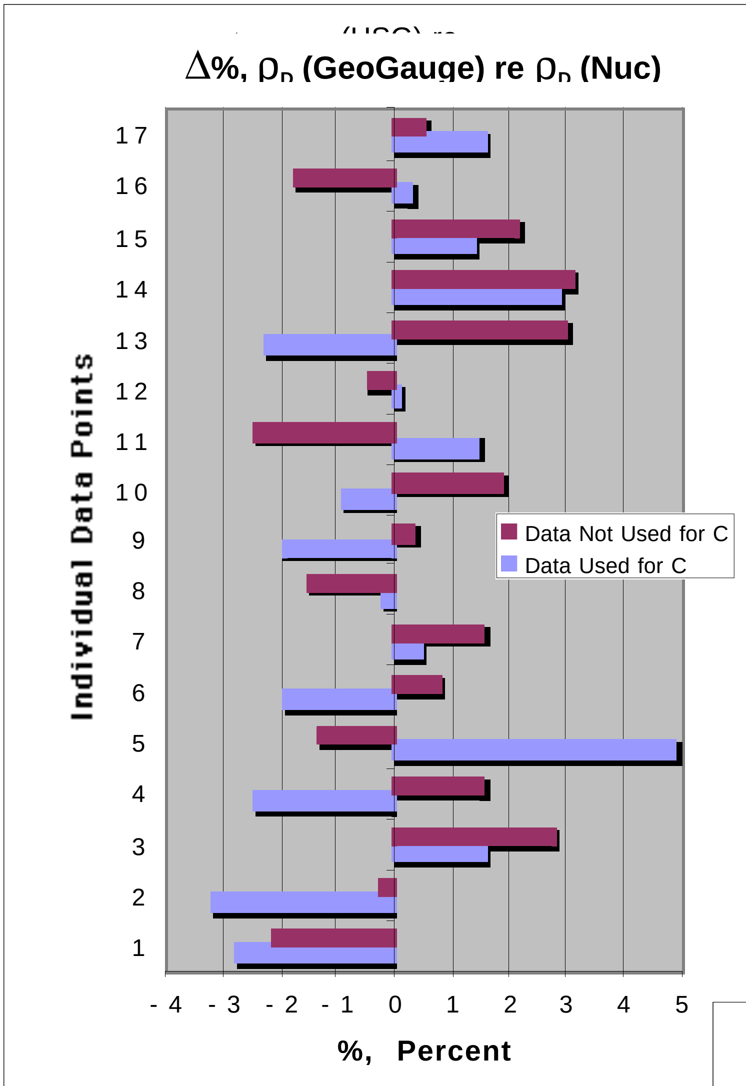
FDOT Raw Lab Data





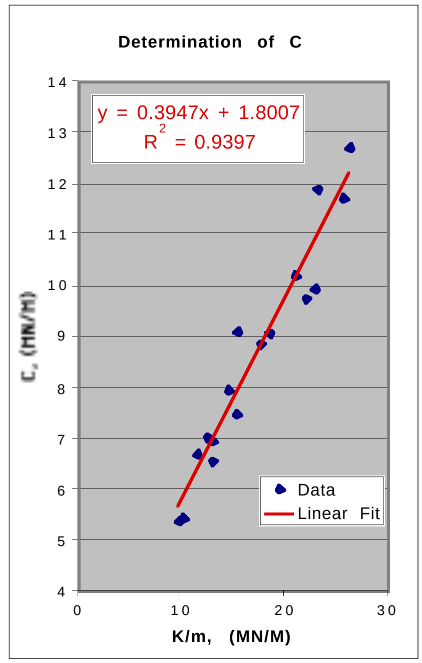


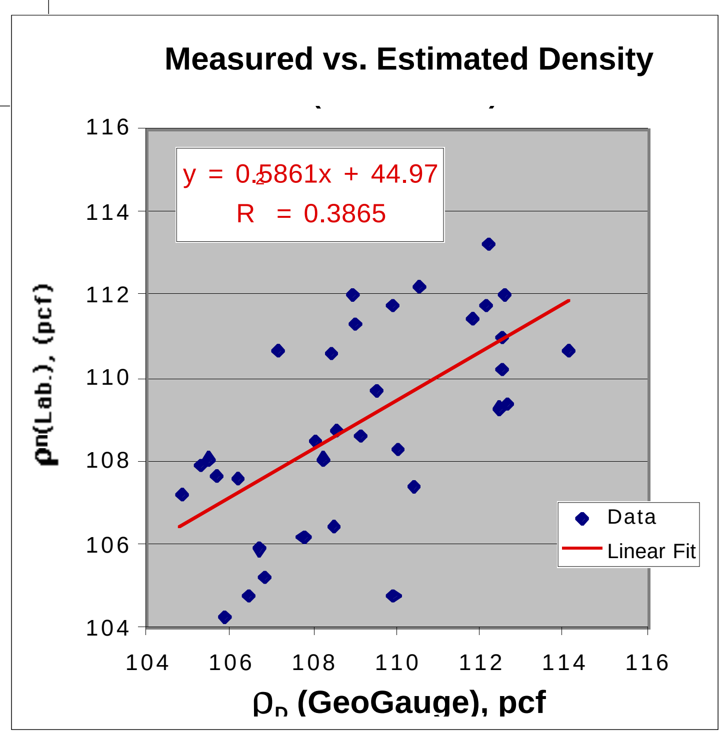
FDOT
Lab Data Analysis Summary



$$\rho_D = \frac{\rho_0}{1 + 1.2\left[\frac{C}{K} - .3\right]^{.5}}$$

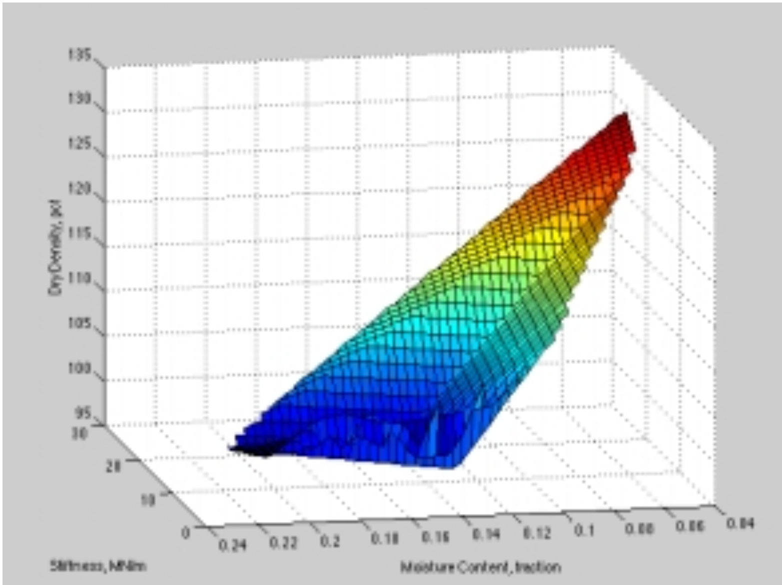
$$C = n(K/m^{.25}) + b$$

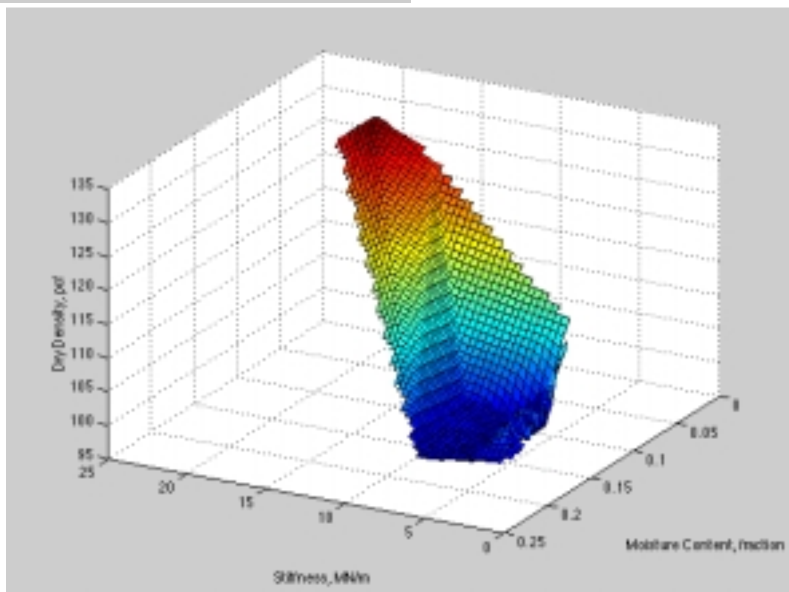


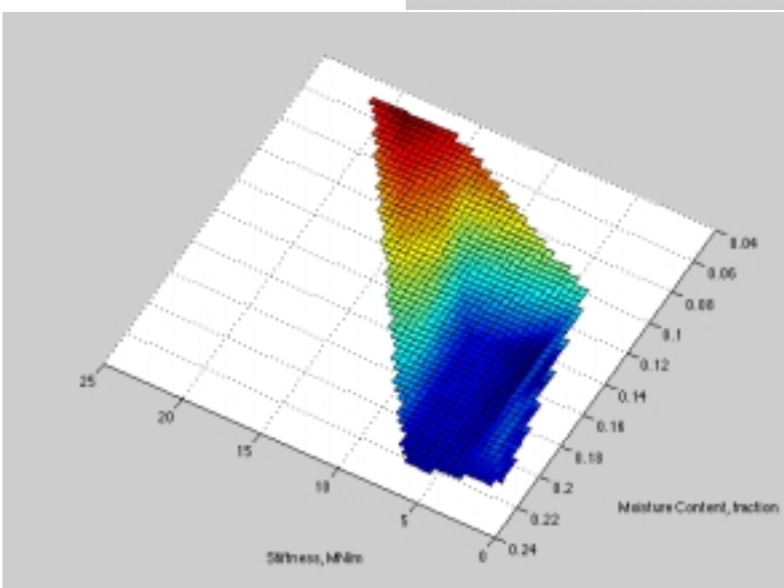


Appendix 7
Analysis of MODOT Data

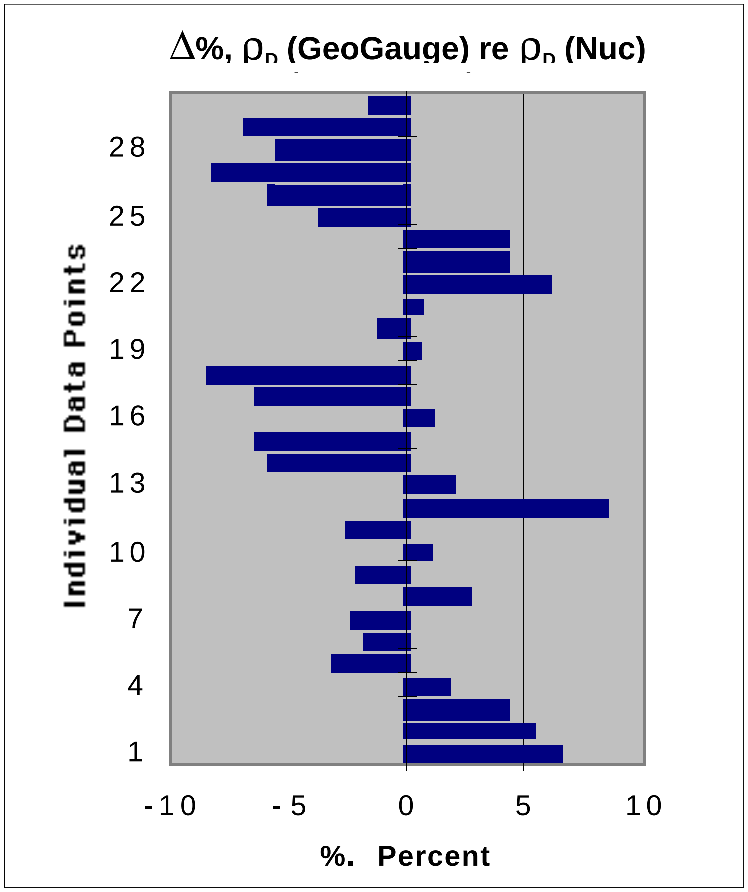
MODOT Raw Data







MODOT
Data Analysis Summary



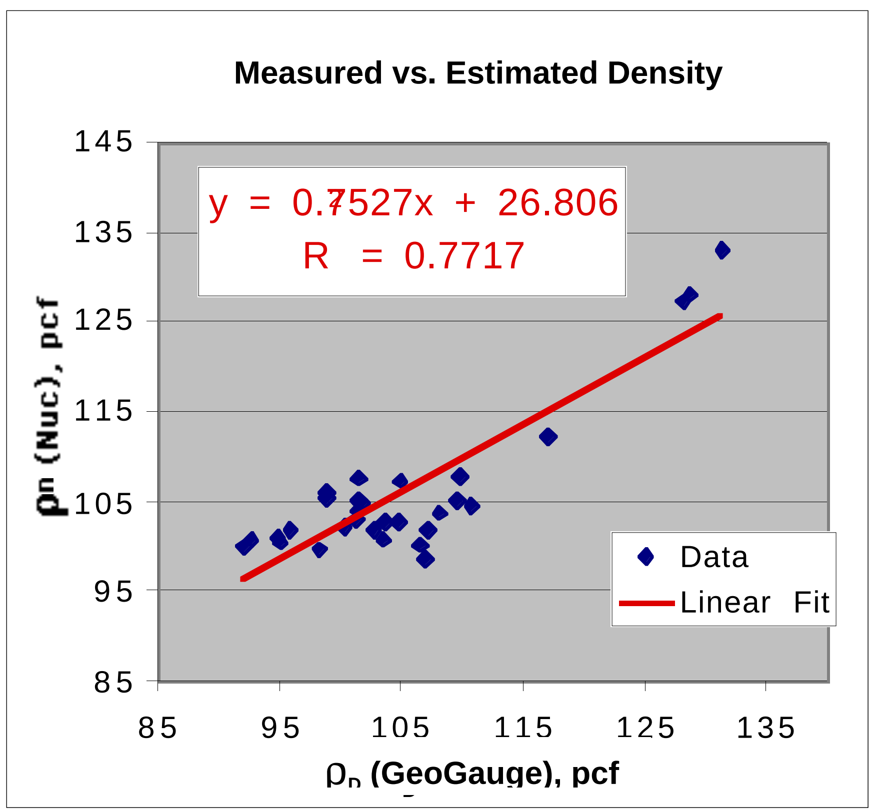
$$\rho_D = \frac{\rho_0}{1 + 1.2\left[\frac{mC}{K} - .3\right]^{.5}}$$

$$C = n(K/m^{.25}) + b$$

Determination of C

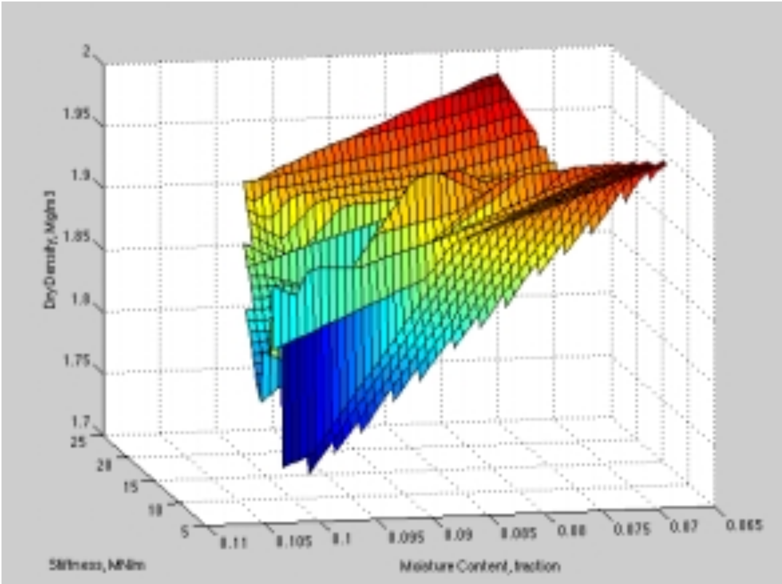
Soil Group #1:
$C = 4.5241(K/m^{.25}) - 2.0602$
$R^2 = 0.9239$

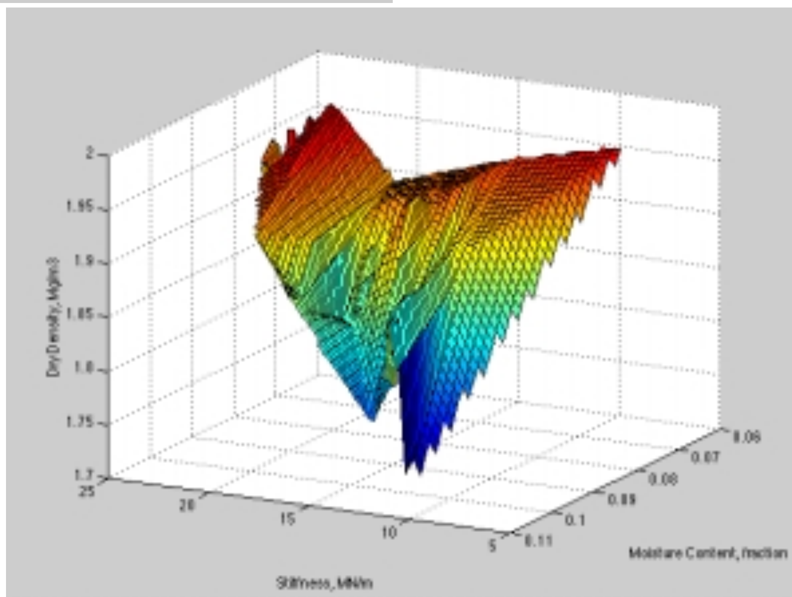
Soil Group #2:
$C = 5.9674(K/m^{.25}) - 16.752$
$R^2 = 0.9834$

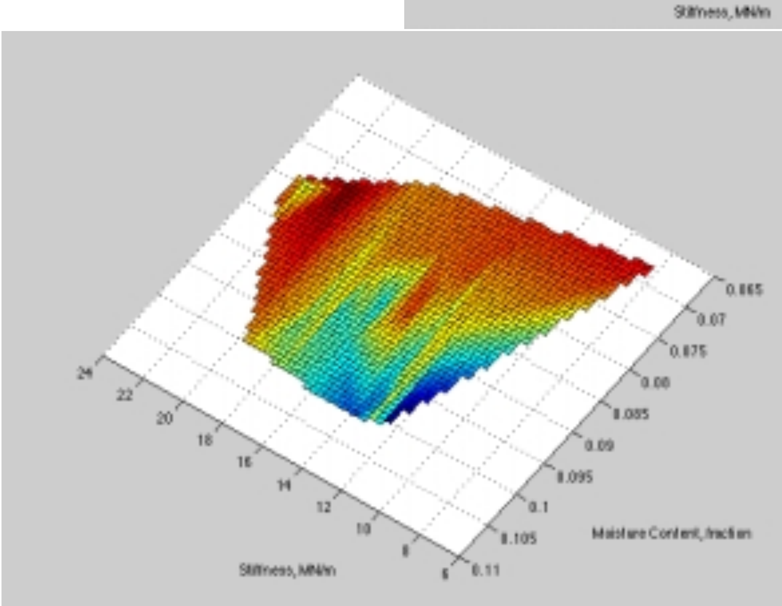


Appendix 8
Analysis of NYSDOT Data

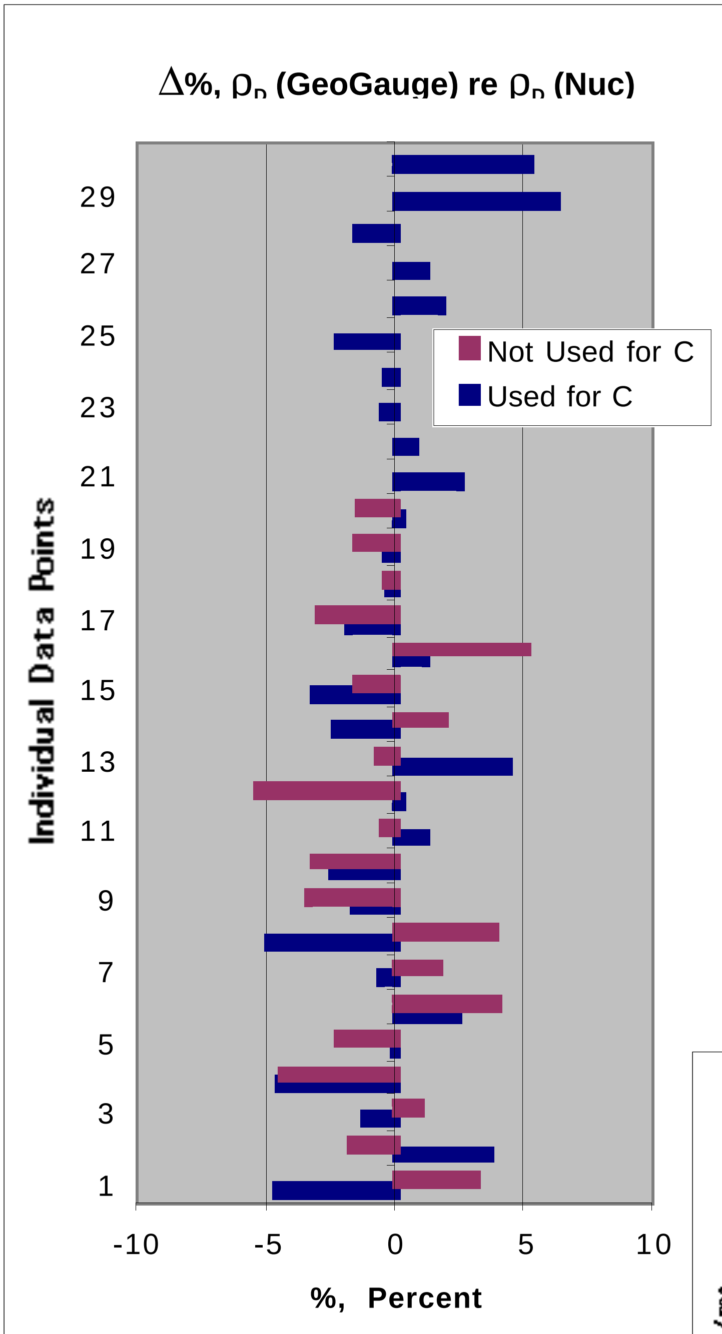
NYSDOT Raw Data





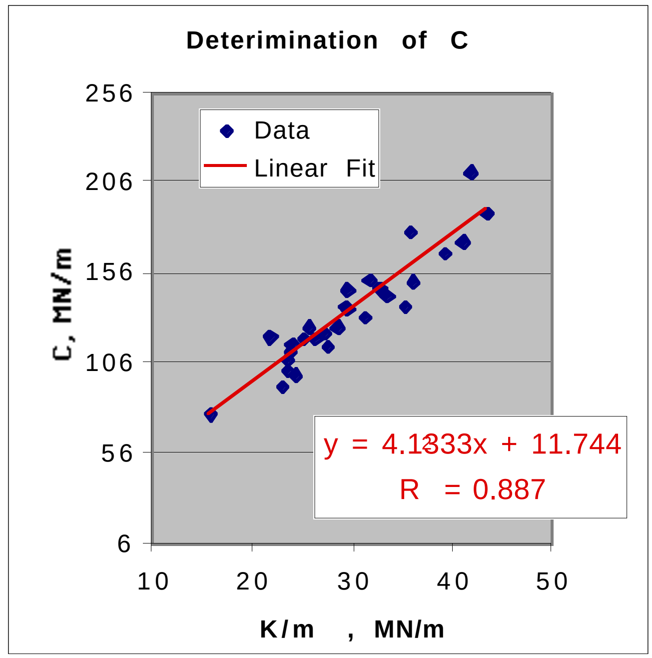


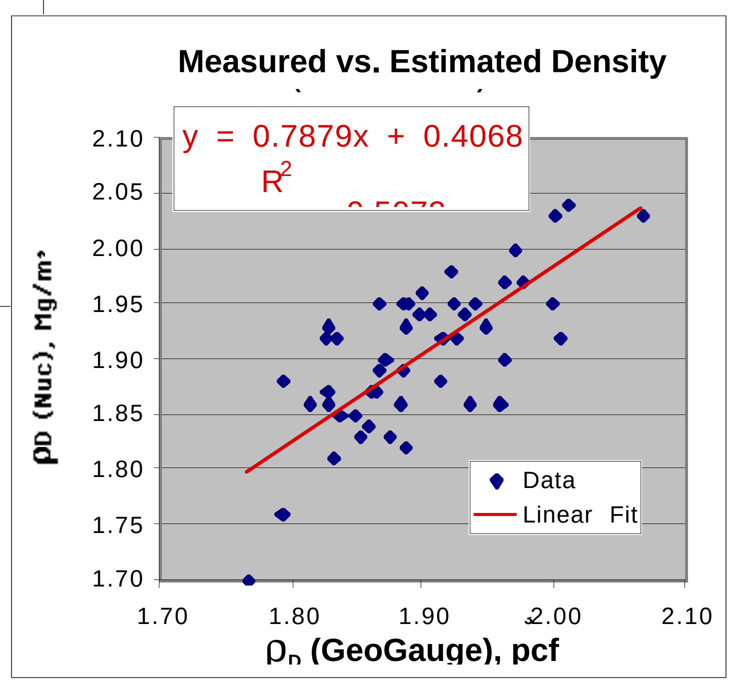
NYSDOT
Data Analysis Summary



$$\rho_D = \frac{\rho_0}{1 + 1.2\left[\frac{mC}{K} - .3\right]^{.5}}$$

$$C = n(K/m^{.25}) + b$$

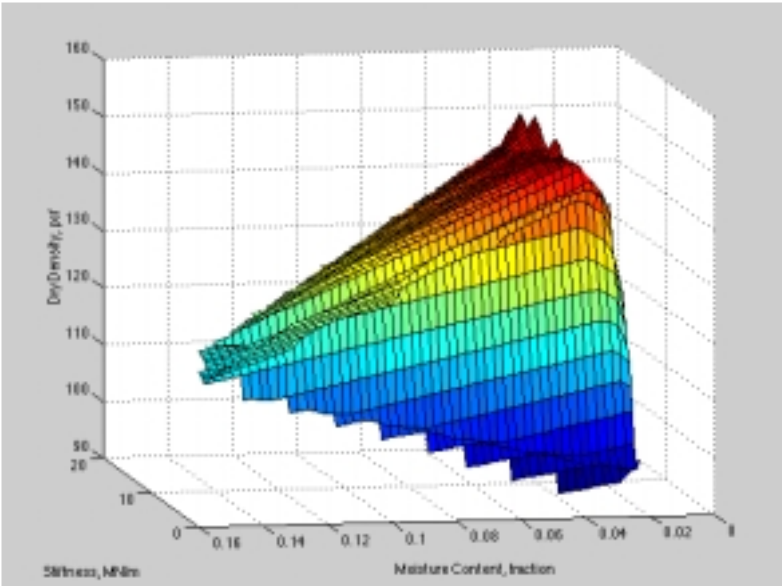


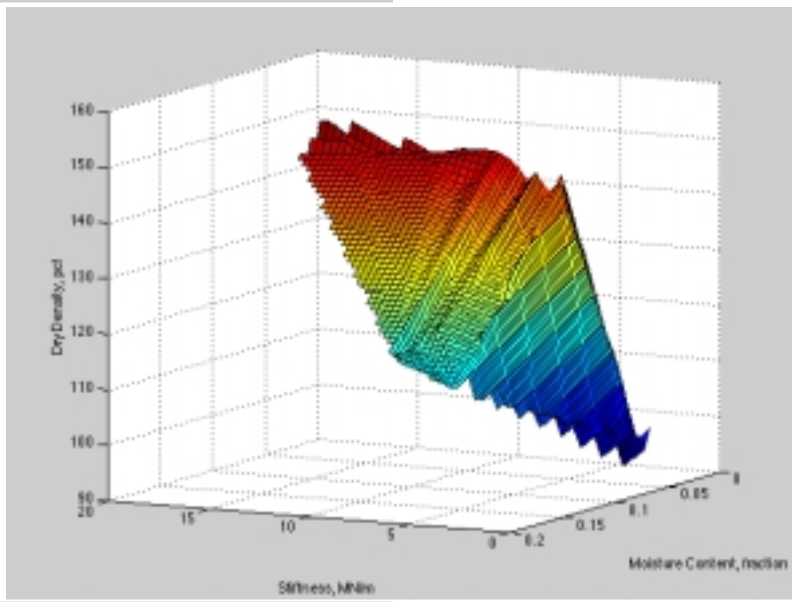


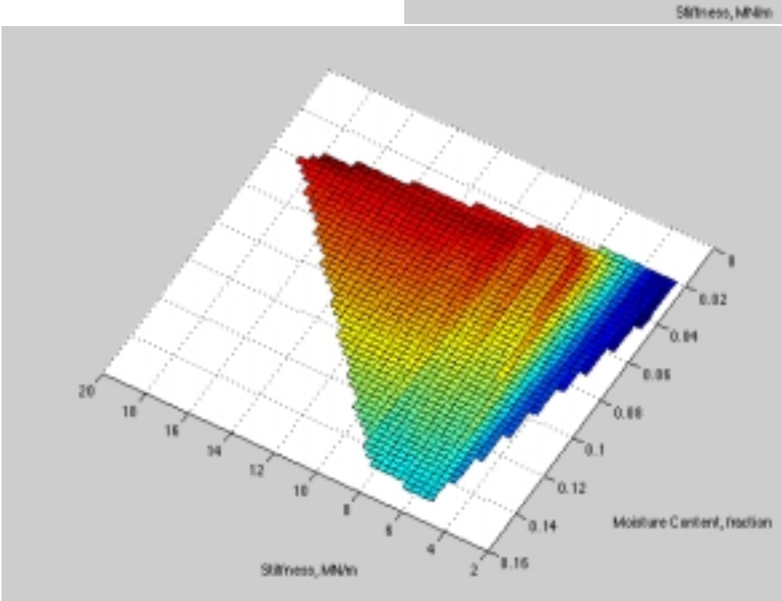
Appendix 9

Analysis of FDOT Lab Data

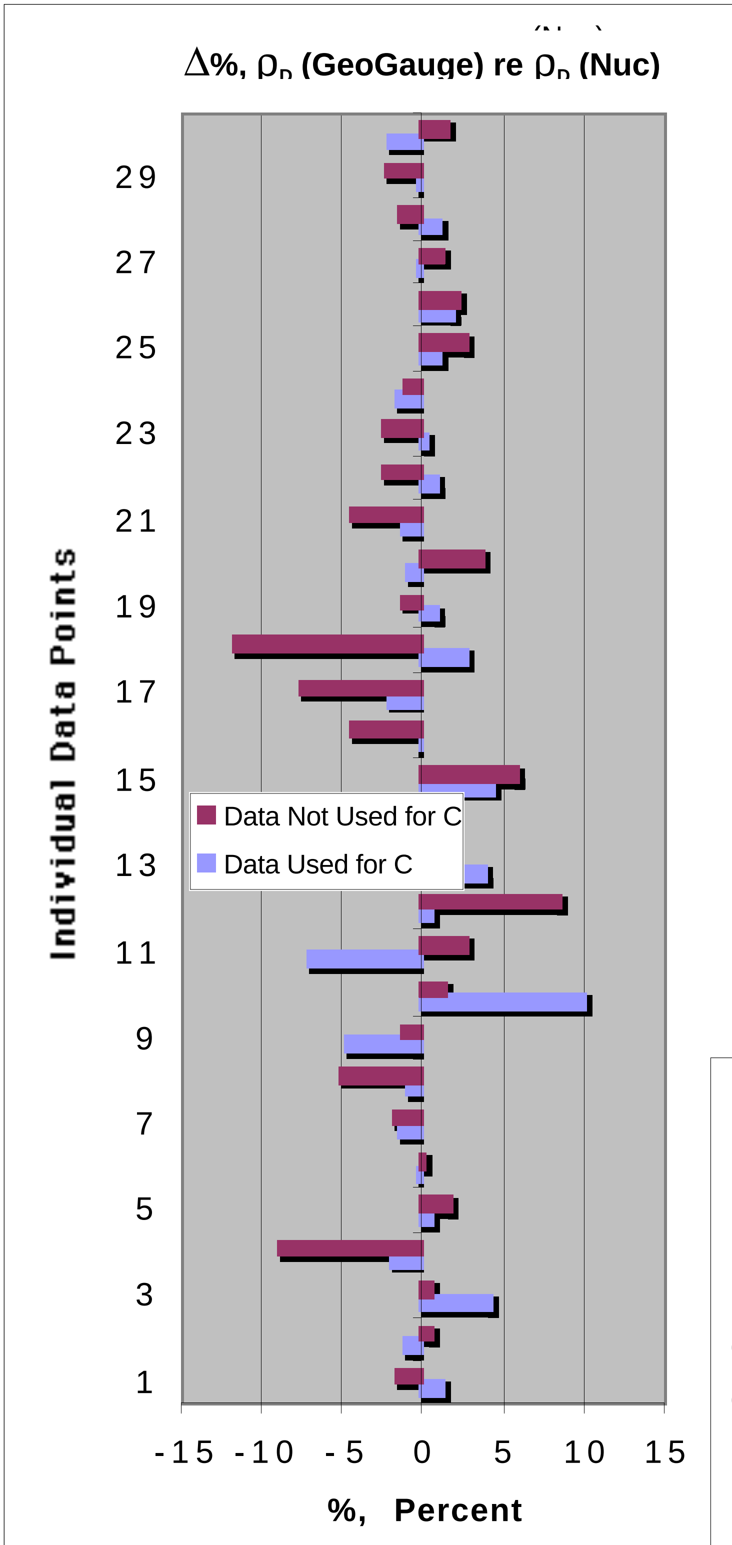
FHWA Raw Data







FHWA Turner- Fairbanks
Data Analysis Summary



$$\rho_D = \frac{\rho_0}{1 + 1.2\left[\frac{mC}{K} - .3\right]^{.5}}$$

$$C = n(K/m^{.25}) + b$$

Determination of C

Soil Group #1 (Bridge):
$C = 7.7675(K/m^{.25}) – 12.337$
$R^2 = 0.5699$

Soil Group #2 (Agg. Pit):
$C = 5.8177(K/m^{.25}) – 25.173$
$R^2 = 0.9839$

Soil Group #3 (Sand Pit):
$C = 3.1862(K/m^{.25}) + 2.5947$
$R^2 = 0.932$

Soil Group #4 (Clay Pit):
$C = 33.626(K/m^{.25}) – 69.423$
$R^2 = 0.9556$